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# Search, Price Setting and Inflation

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A model of monopolistic competition in an inflationary environment is developed which embodies optimal sequential search, price dynamics and entry on the part of consumers and firms respectively. Equilibrium price strategies are  $(S, s)$ ; these bounds increase continuously with consumer search costs, and so does price dispersion. Indeed, the whole equilibrium varies smoothly from the competitive (Bertrand) to the monopolistic (Diamond (1971)) end of the spectrum. The latter's paradoxical result is explained as a limiting case where frictions on firms' side of the market (price adjustment costs) but not on buyers' (search costs) tend to zero. A positive relationship between (smooth and perfectly anticipated) inflation and price dispersion or uncertainty is established.

## INTRODUCTION

Price dynamics in imperfectly competitive markets result from the interplay of sellers' and buyers' strategies. Understanding the microeconomic determinants of price setting and their welfare or macroeconomic implications—such as the role of frictions in monopolistic competition or the effects of inflation—therefore requires an analysis which incorporates the decision problems of both types of agents. With this in mind, this paper brings together two hitherto separated, but highly complementary, strands of the imperfect competition literature, namely optimal price adjustment and search models.

In the literature on price dynamics, sellers, be they monopolistic competitors (Phelps and Winter (1970)), monopolists (Barro (1972), Sheshinski and Weiss (1977), (1982), Rotemberg (1980)), or oligopolists (e.g. Green and Porter (1980), Maskin and Tirole (1985), Gertner (1985), Rotemberg and Saloner (1985), Sheshinski and Weiss (1987)) are endowed with complex optimization problems and sophisticated strategies (optimal control, repeated or Markov games etc.), while the purchasing side of the market is generally oversimplified as an exogenous, instantaneous demand curve.<sup>1</sup> Conversely, the literature on consumer search has mostly ignored price dynamics (important exceptions are Diamond (1971) and Rothschild (1973)), concentrating instead either on characterizing optimal search rules in complex situations (with learning or bargaining, several markets etc.) but where the determination of prices is unspecified, or on obtaining price dispersion in a market equilibrium where price-setting is optimal but static (Rob (1985), Stiglitz (1985), von zur Muehlen (1980)) or follows some *ad hoc* dynamic rule ("experimental behaviour": Axell (1977)).

A more balanced model of a dynamic, imperfectly competitive market is constructed in this paper, with a double objective: to shed light on certain market frictions underlying monopolistic competition, and to provide a theoretical basis for the empirical relationship linking higher inflation rates to increased price dispersion. It uses as an important building block the Sheshinski and Weiss (1977) model, which shows that a monopolist who must keep pace with inflation in the rest of the economy, but faces a fixed cost of changing his price, will optimally follow an  $(S, s)$  real price policy. In other words, he adjusts his nominal price so as to achieve a real value of  $S$  every time this real value has been eroded

down to  $s < S$ . This result was established under two important and related assumptions, both of which substantially restrict the opportunity set of buyers: the first is that the good is not storable (so there is no possibility of buying at today's low real price instead of tomorrow's high real price), the second that there is no competition through customers searching between different firms (hence there is no possibility of buying from a firm charging a low real price instead of one charging a high real price).

Lifting either of these restrictions on substitution by buyers between purchases at different points of the  $(S, s)$  cycle yields important insights into the relationship between inflation and price uncertainty. In Bénabou (1987a), storage is introduced, and the optimal price policy shown to be a stochastic  $(S, s)$  rule, where the bounds are state-dependent random variables, appropriately chosen to try and deter speculation. Inflation—even perfectly regular and anticipated—thus generates price uncertainty at the individual firm level. In the present paper, consumer search is introduced and a causal relationship between inflation and another type of price uncertainty, namely the dispersion of prices which searchers face in the market, is established.

In equilibrium, firms find it optimal—given inflation and consumers' search behaviour—to follow  $(S, s)$  price rules, while consumer search is optimal given the price policies. Inflation generates price dispersion which makes search potentially profitable, thereby increasing competition, and this phenomenon generates many interesting comparative statics and dynamics results. For instance, the bounds  $S$  and  $s$  decrease, and price dispersion between them increases, with the rate of inflation and with price adjustment costs; higher consumer search costs, on the other hand, lead to higher equilibrium prices and allow more price dispersion. Indeed, the whole equilibrium varies continuously from the competitive (Bertrand) to the monopolistic (Diamond (1971)) end of the spectrum; the latter's paradoxical monopoly-price result is easily explained as the limit of a smoother, monopolistically competitive equilibrium—with optimal search, price dynamics and entry—when frictions on firms' side of the market (price adjustment costs) but not on consumers' (search costs) tend to zero.

The model is presented in Section 1, and the existence of a unique equilibrium (satisfying appropriate conditions) established in Section 2. Section 3 examines the effects of frictions (search and adjustment costs) on the equilibrium, while Section 4 focuses on inflation and its relationship to price uncertainty. Finally, Section 5 discusses and relates the model's results to those of the existing literature. Most proofs are given in the appendix.

## 1. THE MODEL

### 1.1. *Tastes and technologies*

*Firms:* A homogeneous good is produced and sold by a continuum of identical and infinitely-lived firms, with discount rate  $r$ . Production (or presence in the market) involves a fixed cost of  $h \geq 0^2$  units of labour per unit of time, and a constant marginal cost of  $c$  units of labour. Firms will thus enter or exit the market until the remaining ones' intertemporal profits (measured in terms of labour) are equal to  $h/r$ . The (endogenous) density of consumers per firm in the market will be denoted as  $x \in (0, +\infty)$ ; an increase in  $x$  corresponds to the exit of firms, a decrease to the entry of new ones.<sup>3</sup> Firms can implement a change in their nominal price at any time, but such a decision is costly: goods must be relabelled, new price lists and catalogues must be printed and sent, etc. Following Barro (1972), Mussa (1976), Sheshinski and Weiss (1977), (1982) and

Rotemberg (1983), it will be assumed that any price adjustment requires a fixed amount  $\beta > 0$  of labour.<sup>4</sup>

*Consumers:* During each interval of time of length  $dt$ , a continuum of consumers with total mass  $1 \cdot dt$  enter the market, with a utility function:

$$U(y, l) = (L - l) + Z \min(y, l)$$

where  $L > Z > c$  is the individual's endowment of labour and  $y$  his consumption of the good, which can be thought of as a durable (only one unit is desired over a lifetime). Consumers do not know how much each seller charges and must therefore search for an acceptable price. Search is instantaneous (or requires a length of time of order smaller than  $dt$ )<sup>5</sup> and consumers cannot postpone their consumption; thus within  $dt$  they all search, buy and exit the market, to be replaced by a similar generation of "instantaneous" consumers an instant later. Costless recall of previous offers is allowed, but since utility is linear and there is no limit to the number of searches a consumer can conduct,<sup>6</sup> optimal search is the same with and without recall, as shown in Lippman and McCall (1976). A first price quotation is received for free,<sup>7</sup> but each subsequent search requires  $\tau$  units of labour, with  $0 < \tau < L - Z$ .

1.2. *Inflation and the distribution of prices*

It will be assumed that the whole economy is on a steady inflationary path: all aggregate nominal quantities, and in particular the nominal opportunity cost of labour (which can be thought of as the wage on a competitive labour market), grow at a rate of  $g > 0$ . All real prices will be expressed in terms of labour. Unlike the cost of labour, any index of the prices charged by firms results from their endogenous strategies; these must therefore replicate, as an aggregate, the inflationary process in response to which they arose (consistent aggregation). Caplin and Spulber (1986) showed that  $(S, s)$  rules possess this important property; Lemma 1 below follows from a more general proposition of theirs:

**Lemma 1.** *If a continuum of price setters follow identical  $(S, s)$  rules with respect to some index inflating at a constant rate  $g$ , the only cross-sectional distribution of their real prices which is invariant over time is log-uniform over  $[s, S]$ . Under this invariant distribution, any index of firms' nominal prices  $P^i$ , which is of the form  $G[\int w(P^i) di]$  and is homogeneous of degree one, grows at the rate of  $g$ .<sup>8</sup>*

In simpler terms, if firms' prices are initially distributed log-uniformly over  $[s, S]$ -equivalently, if their last price adjustment dates are uniformly distributed over  $[-\text{Log}(S/s)/g, 0]$ —this will remain the case forever. The intuition behind these results is simple (for details and formal proofs, cf. Caplin and Spulber (1986)). The logarithm  $\text{Log}(p^i_t)$  of firm  $i$ 's real price at time  $t$  is transformed at time  $t + \delta$  (where  $\delta < T \equiv \text{Log}(S/s)/g$ ) into  $\text{Log}(p^i_{t+\delta}) - g\delta$  if  $\text{Log}(p^i_t) \in (\text{Log}(s) + g\delta, \text{Log}(S)]$ , and into  $\text{Log}(p^i_{t+\delta}) + g(T - \delta)$  if  $\text{Log}(p^i_t) \in [\text{Log}(s), \text{Log}(s) + g\delta]$ . Thus, time simply rotates, at the constant speed  $g$ , firms' log-real prices along the circle of circumference  $gT$  obtained by connecting the extremities of the segment  $[\text{Log}(s), \text{Log}(S)]$ . Such a rotation preserves the uniform distribution over the circle (and only this one), or equivalently the log-uniform distribution of real prices on  $[s, S]$ , and thus keeps constant any average of the type  $\int w(p^i) di$ .

### 1.3. *The equilibrium concept*

The equilibrium of the good's market is a fixed point: individual price rules generate a cross-sectional distribution, which in turn determines optimal search, hence demand and thereby optimal price strategies; finally, the density of participating firms must leave each of them with zero profits. More formally, an equilibrium will be defined by the following four conditions:

- E1. Symmetric  $(S, s)$  Nash equilibrium in price strategies: given that all other firms follow a common  $(S, s)$  rule, and given consumers' search strategy, a firm's optimal price policy is that same  $(S, s)$  rule.
- E2. Steady-state distribution: real prices are initially distributed over  $[s, S]$  according to the invariant distribution (log-uniform).
- E3. Optimal sequential search: consumers search optimally, given firms' price strategies.
- E4. Free entry: each firm's real intertemporal profits are zero.

It should be noted that E1 only restricts attention to a specific class of equilibria (symmetric  $(S, s)$ ), but within the most general price strategy space. As to condition E2, it is justified (or even required) by the following three arguments of optimality, macro-economic consistency, and stability. As indicated by Lemma 1, the invariant distribution is the only one which is preserved over time by the combination of inflation and  $(S, s)$  policies. Any other initial conditions will lead to a time-varying distribution, generally resulting in non-stationary search and demand, which in turn destroy the optimality of an  $(S, s)$  rule. Even if such is not the case, a time-varying distribution of real prices implies that any aggregate index of firms' nominal prices does not grow at the constant rate  $g$  (in particular, if all firms start at the same price, they remain synchronized and any such index is discontinuous); thus, even if the market by itself can be in such an equilibrium, it cannot be a consistent component of a smoothly inflating economy. Finally, for any initial cross-sectional distribution of real prices, convergence to the steady-state distribution occurs (exponentially) if the bounds  $(S, s)$  differ slightly between firms (as mentioned in Caplin and Spulber (1986)), or are randomized so as to limit storage by speculators (as in Bénabou (1987a)).<sup>9</sup> Related results arise in the literature on time-contingent price (or wage) adjustment (e.g. Fethke and Policano (1986), Ball and Romer (1987)), showing that firm-specific shocks tend to make staggered (respectively, synchronized) price setting stable (respectively, unstable).

## 2. THE EQUILIBRIUM

The distribution of real prices resulting from E2 is:

$$(\forall p \in [s, S]) \left( d\mu(p) = \frac{dp/p}{\text{Log}(S/s)} \right). \quad (1)$$

It will be convenient to measure the dispersion of prices in the market by  $(S-s)/s = 1/\sigma - 1$ , where  $1/\sigma = S/s$  is the ratio of the highest to the lowest price, which is here equivalent to the standard measure of dispersion:

**Lemma 2.1.** *The ratio  $(\text{Var}[p])^{1/2}/E[p]$  of the distribution  $d\mu$ 's standard deviation to its mean depends only on  $1/\sigma = S/s$ , and is a strictly increasing function of this variable.*

*Proof.* See appendix.  $\parallel$

2.1. *Search and demand*

The equilibrium distribution  $d\mu$  of real prices in the market is constant over time and known to searchers; optimal sequential search is therefore determined by a real cutoff price  $R$ : a consumer keeps searching until he encounters a real price no greater than  $R$ , where.<sup>10</sup>

$$R = \tau + E[\min(R, p)]$$

which expresses indifference between stopping when  $R$  is offered and doing one more search, with the possibility of coming back to  $R$  if a higher price is encountered. Equivalently:

$$\tau = E[R - \min(R, p)] = \int_s^{\min(R, S)} (R - p) d\mu(p). \tag{2}$$

A consumer's purchasing decision is thus characterized by his effective reservation price  $Q = \min(Z, R)$ , which is the minimum of the two cutoff prices  $Z$  and  $R$ , derived respectively from preferences and optimal search: he buys if and only if  $p \leq \min(Z, R)$ . Since consumers are identical, they have the same  $Z$ ,  $R$  and  $Q$ . Moreover, when sampling stores, they do so at random, so that each firm is visited by the same number of searchers, and (the density of) demand per firm is therefore:

$$d(p) = x \cdot \mathbb{1}_{\{p \leq \min(Z, R)\}} \tag{3}$$

where  $\mathbb{1}_{\{\cdot\}}$  denotes the indicator function, equal to 1 if the inequality between brackets holds, and to zero otherwise.

2.2. *The firm's optimization problem*

Together with adjustment costs, this stationary demand function determines the optimal price policy of individual firms (which take  $Q$  and  $x$  as given).<sup>11</sup>

**Theorem 2.1.** *Assume that  $\beta \leq Qx/(r + g)$ . The optimal price policy of a representative firm is an  $(S, s)$  rule with  $S = Q$  and  $s/S = \sigma^*(r, g, \beta/Qx)$ , determined as the unique solution to:*

$$rV = x(s - c) = x(Q\sigma - c)$$

where  $V$  denotes real intertemporal operating profits, equal to:

$$V = \frac{\int_{\sigma}^1 x(Qu - c)u^{-1+r/g} du - \beta g}{g(1 - \sigma^{r/g})}$$

Moreover,  $\sigma^*$  is continuously differentiable in all its arguments, and decreasing in  $\beta/Qx$  (resp. in  $g$ ) from a limit of 1 (resp. of  $1 - r\beta/Qx$ ) at zero. The firm operates and implements this policy if and only if  $rV \geq h$ , or:  $\sigma^*(r, g, \beta/Qx) \geq (c + h/x)/Q$ .

*Proof.* See appendix. ||

An alternative parametrisation of  $s$  and  $V$  is provided by the periodicity of price adjustments  $T \equiv \text{Log}(S/s)$ :

$$V = \frac{\int_0^T x(Qe^{-gt} - c)e^{-rt} dt - \beta}{(1 - e^{-rT})}$$

It is never profitable to overshoot the real reservation price  $Q$ : the adjustment cost would be postponed, but so would the whole path of net future revenues, which is larger ( $V \geq h/r$ ). The optimality condition  $x(s - c) = rV$  is also quite intuitive: delaying adjustment by  $dt$  brings a marginal revenue of  $x(s - c)dt$ , but the maximum valuation is achieved only after  $dt$ , hence an opportunity cost of  $rVdt$ . Because of the adjustment cost, the real intertemporal profits resulting from the optimal policy are the same as those the firm would earn if it sold the good at its minimum real price of  $s$  forever ( $V = x(s - c)/r$ ).

It should also be noted that when the inflation rate  $g$  converges to zero, the optimal time between adjustments  $T^*(r, g, \beta/Qx)$  goes to  $+\infty$ , as suggested by intuition, but the ratio  $s/S = \sigma^*(r, g, \beta/Qx)$  remains bounded away from 1. This is in fact a general, but previously unnoticed, feature of  $(S, s)$  models: for the seller to be willing to change his price, the increase in revenue gained by adjusting from  $s$  to  $S$  must be large enough to compensate for the discrete cost of adjustment. The implications of this phenomenon will be examined in Section 4.

Finally, in equilibrium, both  $Q$  and  $x$  are endogenous and influence each firm's price dynamics through the size of its sales relative to its adjustment cost ( $\beta/Qx$ ). In particular, there is a crowding externality among firms: a lower  $x$  (entry) causes a more than proportional decrease in profits, because the optimal  $s$  also decreases, as a small market share is not worth frequent price adjustments (cf. Theorem 2.1).<sup>12</sup>

The existence and uniqueness of the equilibrium will now be established in two steps, to which the next two subsections correspond. First, implicit equations characterizing the two possible equilibrium configurations of  $S$  are derived. These are then brought together with the conditions expressing free entry and the optimality of individual price adjustments into a single implicit equation, which is finally solved.

### 2.3. Characterization of the equilibrium

The density  $x$  of consumers and their effective reservation price  $Q$  determine firms' optimal price dynamics (cf. Theorem 2.1); the resulting bounds  $(S, s)$  in turn characterize the invariant price distribution which consumers use to compute their search cutoff price  $R$ , and finally  $Q = \min(R, Z)$ . This closes the fixed-point loop. Moreover, it is clear that an equilibrium can be one of two types, depending on whether price-setters' monopolistic power is effectively constrained by preferences ( $Q = Z$ ) or by competition through search ( $Q = R$ ).

#### *A-Binding preferences:*

Assume first that  $Z \leq R$ . By Theorem 2.1,  $S = Z$  and  $s = Z\sigma^*(r, g, \beta/Zx)$ . In such an equilibrium, firms are able to adjust their price to the maximum value  $S = Z$  permissible by consumers' intrinsic willingness to pay for the good. This maximum price is below

consumers' optimal cutoff price from search,  $R$ , so that no one engages in search. The optimal stopping rule (2) defining  $R$  becomes:

$$R = \tau + E[\min(R, p)] = \tau + E[p] = \tau + \int_s^S p d\mu(p)$$

$$= \tau + \frac{S-s}{\text{Log}(S/s)} = \tau + \frac{Z(1-\sigma)}{\text{Log}(1/\sigma)}$$

which by assumption must be no smaller than  $Z$ , so that:

$$Z \leq \frac{\tau}{1-(1-\sigma)/\text{Log}(1/\sigma)} \tag{4a}$$

Although this inequality implicitly depends on  $x$  and even  $Z$  (because  $\sigma = \sigma^*(r, g, \beta/Zx)$ ), it gives the right intuition: preferences are binding in equilibrium when search costs are sufficiently large with respect to  $Z$ , and when price dispersion in the market ( $1/\sigma$ ), which makes search attractive, is not too important (the R.H.S. of (4a) is increasing in  $\sigma$ ). Note that for a given maximum price  $S$ , price dispersion  $1/\sigma$  and the average real price  $(S-s)/\text{Log}(S/s) = S(1-\sigma)/\text{Log}(1/\sigma)$  are negatively related, so that the two search-inducing effects of a lower average price and increased dispersion coincide here.

**B-Binding search**

Assume now that  $Z > R$ . By Theorem 2.1,  $S = R$  and  $s = S\sigma^*(r, g, \beta/Sx)$ . Firms can only adjust to a maximum real price  $S = R < Z$ , because any higher price would trigger search and result in zero demand. The optimal stopping rule now becomes:

$$S = \tau + E[p] = \tau + \frac{S-s}{\text{Log}(S/s)}$$

or

$$S = \frac{\tau}{1-(1-\sigma)/\text{Log}(1/\sigma)} < Z. \tag{4b}$$

Again,  $S$  and  $x$  are implicit in the middle term (because  $\sigma = \sigma^*(r, g, \beta/Sx)$ ), but (4b) correctly suggests that search is binding in equilibrium when search costs are not too large with respect to  $Z$ , and there is sufficient price dispersion in the market. For all positive  $S$  and  $x$ , denote:

$$\frac{1}{\theta(r, g, \beta/Sx)} \equiv 1 - \frac{1-\sigma^*(r, g, \beta/Sx)}{\text{Log}(1/\sigma^*(r, g, \beta/Sx))}$$

Then (4a) and (4b) can be summarized as:

$$S = \min \{Z, \tau\theta(r, g, \beta/Sx)\}. \tag{4c}$$

**2.4. Existence and uniqueness**

In equilibrium, each firm chooses its minimum price  $s = S\sigma$  (or equivalently, the timing



of its adjustments) optimally, equating (by Theorem 2.1):

$$s = c + \frac{rV}{x} = \frac{rS \left[ \int_{\sigma}^1 u^{r/g} du - \beta g / Sx \right]}{g(1 - \sigma^{r/g})},$$

or

$$\sigma = \frac{r[1 - \beta(r+g)/Sx - \sigma^{1+r/g}]}{(r+g)(1 - \sigma^{r/g})}. \quad (5)$$

Moreover, entry or exit has taken place until the remaining firms' profits  $V - h/r = (x(s-c) - h)/r$  are equal to zero, or:

$$s = c + h/x \quad (6)$$

Using (6) and  $S = s/\sigma$  to eliminate  $S$  from condition (5) which expresses the *optimal timing* of price adjustments yields:

$$\Phi(\sigma, x) \equiv (r+g)\sigma - g\sigma^{1+r/g} - r \left[ 1 - \frac{(r+g)\beta\sigma}{cx+h} \right] = 0. \quad (7)$$

Similarly, (4c) which determines the type of the equilibrium, becomes:

$$\sigma = \max \left[ \frac{c+h/x}{Z}, \frac{c+h/x}{\tau\theta} \right]. \quad (8)$$

By definition of  $\theta$ ,  $\sigma = (c+h/x)/\tau\theta$  if and only if:

$$\tau\sigma = \left( c + \frac{h}{x} \right) \left( 1 - \frac{1-\sigma}{\text{Log}(1/\sigma)} \right),$$

so that *search is binding* in equilibrium when:

$$\Psi(\sigma, x) \equiv \frac{1-\sigma}{\text{Log}(1/\sigma)} + \frac{\tau\sigma}{c+h/x} - 1 = 0. \quad (9)$$

The solutions to the implicit equations (7) and (9) will now be examined.

**Lemma 2.2.** *For all  $x > 0$ , there exists a unique  $(\sigma_0(x), \sigma_1(x))$  in  $(0, 1)^2$  such that  $\Phi(\sigma_0(x), x) = 0$  and  $\Psi(\sigma_1(x), x) = 0$ . Moreover,  $\sigma_0$  is strictly increasing, and  $\sigma_1$  strictly decreasing, in  $x$ .*

*Proof.* See appendix. ||

Since  $\partial\Psi(\sigma, x)/\partial\sigma > 0$  (cf. proof of the Lemma), preferences are binding, i.e.  $\sigma > (c+h/x)/\tau\theta$ , or  $\Psi(\sigma, x) > 0$ , if and only if  $\sigma > \sigma_1(x)$ . The equilibrium conditions (7) and (8) are therefore equivalent to:

$$\sigma = \sigma_0(x) \quad (10)$$

$$\sigma = \max \{ \sigma_1(x), (c+h/x)/Z \} \quad (11)$$

or finally, defining:  $\sigma_2(x) \equiv (c+h/x)/Z$  and eliminating  $\sigma$ :

$$\sigma_0(x) = \max \{ \sigma_1(x), \sigma_2(x) \}. \quad (12)$$

The curves  $\sigma_2$ ,  $\sigma_1$  and  $\sigma_0$  are plotted on Figure 1. Since (6) was incorporated into each of them, they are iso (zero) profit lines, along which  $rV = h$ .

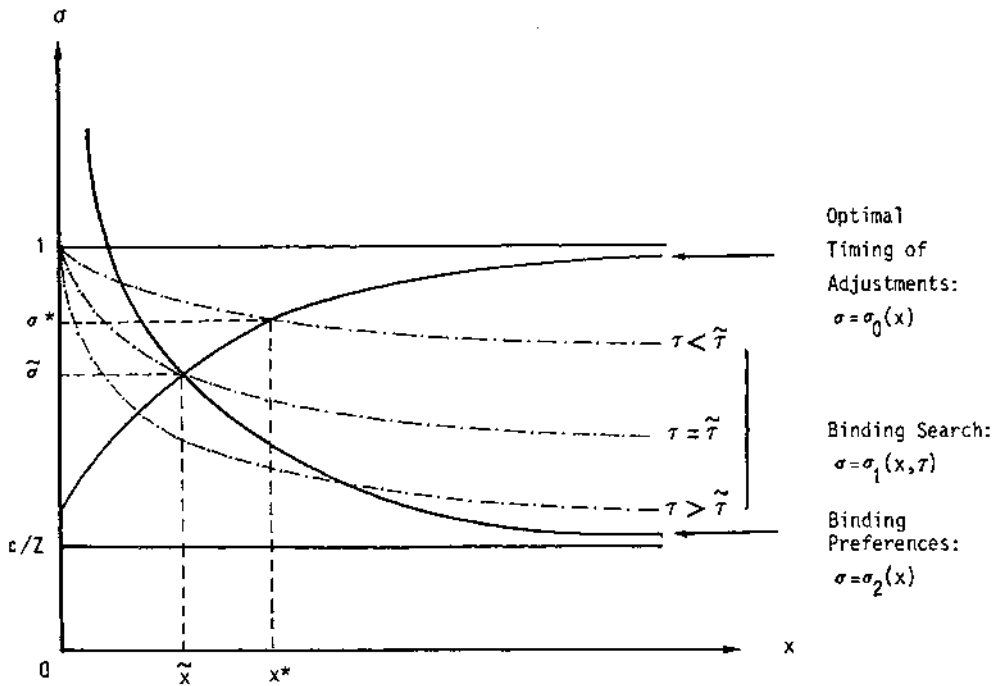


FIGURE 1  
Determination of the equilibrium

The  $\sigma_0$  curve represents the optimal periodicity of price adjustments: entry (a lower  $x$ ) increases a firm's adjustment cost relative to its market share ( $\beta/Sx$ ), making less frequent adjustment—i.e. a lower  $\sigma$ -optimal; the curve is therefore upward sloping. The  $\sigma_2$  curve represents the binding preferences condition ( $S = Z$ ): for profits to remain constant in spite of entry (a decrease in  $x$ ), the lower bound  $s = c + h/x$  must increase; thus  $\sigma = s/Z$  must also increase, so that  $\sigma_2$  is downward-sloping. Finally,  $\sigma_1$  represents the binding search condition ( $S = \tau\theta$ ). Here again, entry's depressing effect on profits must be compensated by a higher  $s$ , here equal to  $\tau\theta\sigma$ . Hence  $\sigma$  must increase ( $d\theta/d\sigma > 0$ ), and  $\sigma_1$  also slopes downwards: more frequent adjustments reduce price dispersion and the potential for search, allowing firms to charge higher maximum  $S = \tau\theta$  and minimum  $s = \tau\theta$  real prices.

An equilibrium is an intersection  $(x^*, \sigma^*)$  of the increasing  $\sigma_0$  and decreasing  $\sigma_3 \equiv \max(\sigma_1, \sigma_2)$ , of which there always is one and only one. Moreover, search is binding if and only if  $\sigma_0$  cuts  $\sigma_1$  above  $\sigma_2$ . Let  $(\tilde{x}, \tilde{\sigma})$  be the intersection of  $\sigma_0$  and  $\sigma_2$  (equilibrium with binding preferences), and define:

$$\tilde{\tau} \equiv Z \left[ 1 - \frac{1 - \tilde{\sigma}}{\text{Log}(1/\tilde{\sigma})} \right]. \tag{13}$$

Thus  $\tilde{\tau}$  is the unique value for which the  $\sigma_1$  curve cuts  $\sigma_0$  and  $\sigma_2$  at their common point  $(\tilde{x}, \tilde{\sigma})$  (cf. Figure 1), i.e. such that  $S = Z = R$ . As  $\tau$  decreases below  $\tilde{\tau}$ , the  $\sigma_1$  curve pivots up and cuts  $\sigma_0$  above  $\sigma_2$ , so search is and becomes increasingly binding, with  $\sigma^*$  and  $x^*$  increasing (less price dispersion, exit of firms). As  $\tau$  increases above  $\tilde{\tau}$ , on the other hand, the  $\sigma_1$  curve pivots down, cutting  $\sigma_0$  below  $\sigma_2$ ; hence the solution  $x^*$  to  $\sigma_0(x) = \sigma_3(x)$  remains equal to  $\tilde{x}$  and  $\sigma^*$  to  $\tilde{\sigma}$  (binding preferences). More formally:

**Theorem 2.2.** *There exists a unique market equilibrium, with a density  $1/x > 0$  of firms following a common  $(S, s)$  pricing rule and earning zero profits. The real price bounds satisfy:*

$$S = \min \left\{ Z, \frac{\tau}{1 - (S - s)/\text{Log}(S/s)} \right\}; \quad s/S = \sigma^*(r, g, \beta/Sx)$$

with the function  $\sigma^*$  defined as in Theorem 2.1. Moreover, there exists  $\tilde{\tau} > 0$  such that search is binding in equilibrium ( $S < Z$ ) if and only if  $\tau < \tilde{\tau}$ .

*Proof.* See appendix. ||

The equilibrium is represented as a function of  $\tau$  in Figures 2(a) and 2(b). When search costs are large ( $\tau \geq \tilde{\tau}$ ), searching is not worthwhile, and only inflation combined with adjustment costs prevents sellers from enjoying their full monopoly power ( $S = Z$ ). When search costs are small, on the contrary ( $\tau < \tilde{\tau}$ ), a new type of equilibrium emerges, in which the threat of consumer search effectively *imposes competition* among firms: at the price  $S = \tau\theta$ , consumers are just indifferent between buying and searching further, so that any adjustment to a higher real price—such as the monopoly level  $Z$ —would result in zero demand.

### 3. MARKET FRICTIONS AND THE DIAMOND PARADOX

A significant part of the search literature has been devoted to obtaining equilibrium price dispersion. This involves getting around the rather counterintuitive result (Diamond (1971)) which is generated by the basic search model where there are identical firms and consumers: if search is free, all firms must charge the competitive price, but if it is not, the unique equilibrium is for all firms to charge the monopoly price, no matter how high it is, how small the search cost, and how many firms there are.<sup>13</sup> As described by Stiglitz (1985), “search costs, even small search costs, have an enormous effect on the nature of the equilibrium”. This result contradicts the economic intuition of “the dependence of equilibrium prices on the abilities of traders to find alternatives” (Diamond (1987a)), which in the present context means that equilibrium prices should be continuous and decreasing functions of search costs. The following result (illustrated in Figures 2(a) and 2(b)) shows that as soon as even very small amounts of inflation and price adjustment costs are introduced, the equilibrium acquires this fundamental property.

**Proposition 3.1.** *The equilibrium is a continuous, monotonic and differentiable (except at  $\tilde{\tau}$ ) function of the search cost  $\tau$ . As  $\tau$  decreases from  $\tilde{\tau}$  to zero, the real price bounds  $(S, s)$  decrease from their monopoly levels  $(Z, Z\tilde{\sigma})$  to the competitive price  $c$ , while the periodicity of price adjustment, price dispersion and the density  $1/x$  of firms in the market all decrease to zero.*

*Proof.* See appendix. ||

This result is particularly interesting because it shows how the monopolistically competitive equilibrium with price dispersion becomes increasingly competitive as search costs decrease, covering a spectrum which ranges from the purely monopolistic case (*Diamond*) to the purely competitive one (*Bertrand*): the mounting competitive pressure generated by search forces sellers to charge real prices which are both lower and more in line with one another, and reduces the number of firms which can profitably operate

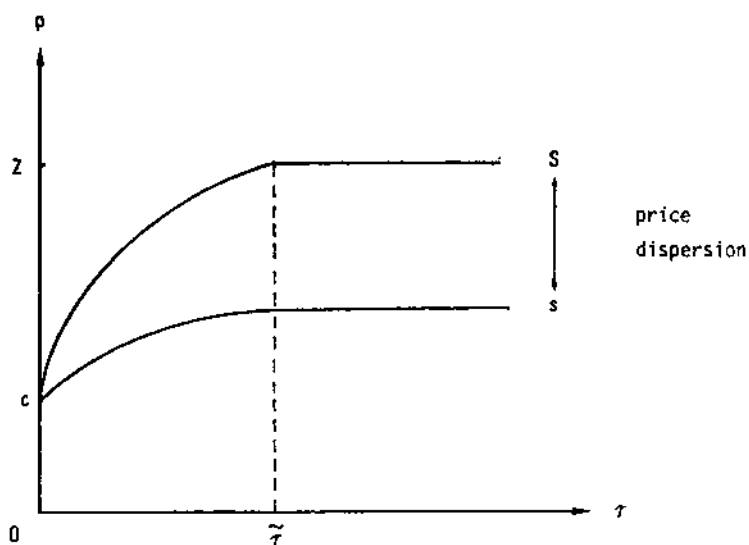


FIGURE 2(a)  
The effect of search costs on real prices

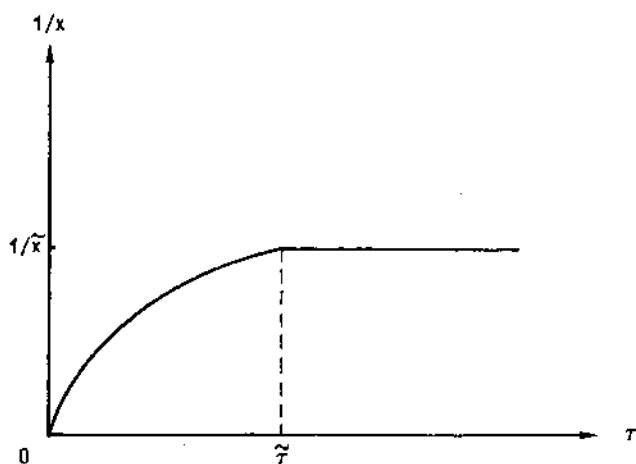


FIGURE 2(b)  
The effect of search costs on the density of firms in the market

in the market. In the limit, a discrete number of firms with negligible fixed costs (compared to market size:  $h/x = \beta/x = 0$ ) charge the marginal cost. The presence of frictions in the functioning of the market on the firms' side (costly price adjustment) as well as the consumers' (costly search) thus restores the balance which the original model lacked and ensures a smoother, more realistic outcome. In this more symmetric light, Diamond's result is easily understood as one of the two polar limiting cases where frictions on one side vanish: if search is free but price adjustment costly, the market is totally biased against sellers and the competitive price prevails (cf. Proposition 3.1); if price adjustment is free but search costly, the market is totally biased against buyers and all purchases take place at the monopoly price:

**Proposition 3.2.** *The equilibrium is a continuous and monotonic function of the adjustment cost  $\beta$ , and there exists  $\tilde{\beta} > 0$  such that search is binding if and only if  $\beta > \tilde{\beta}$ ; the equilibrium is differentiable except at  $\tilde{\beta}$ . As  $\beta$  decreases to zero, the real price bounds  $(S, s)$  increase to the monopoly price  $Z$ ; price dispersion and the periodicity of price adjustment decrease to zero, while the density  $1/x$  of firms increases to the maximum value  $(Z - c)/h$  sustainable by the market.*

*Proof.* See appendix. ||

This proposition is illustrated in Figures 3(a) to 3(c).<sup>14</sup> The first two depict the equilibrium as a function of  $\beta$ , for a given value of  $\tau$ ; the last one illustrates how the continuous function, associating to any  $\tau \geq 0$  the corresponding average price in the market  $E[p]$ , converges from below to the discontinuous static equilibrium function  $[\tau \rightarrow c + (Z - c)\mathbb{1}_{\{\tau > 0\}}]$  as  $\beta$  tends to zero (the graphs for  $S$  and  $s$  would be similar).

These results are quite intuitive: as it becomes less costly to change prices, firms do it more frequently, thereby reducing (for any given maximum real price) the amount of price dispersion and raising the average price in the market; this in turn decreases consumers' willingness to search, allowing firms to adjust to a higher maximum level and generating entry. When prices become perfectly flexible, all equilibrium prices converge to the monopoly level  $Z$  (as long as  $\tau > 0$ ) and search disappears altogether, so that the Diamond (1971) result is obtained in the limit.<sup>15</sup> The limiting density of firms is that which leaves each of them with profits of  $((Z - c)x - h)/r = 0$  when all charge the monopoly price  $Z$  forever.

#### 4. INFLATION, PRICE DISPERSION AND COMPETITION

There is considerable evidence that higher rates of inflation—even perfectly regular and anticipated ones—are associated with (and probably the cause of) greater relative price

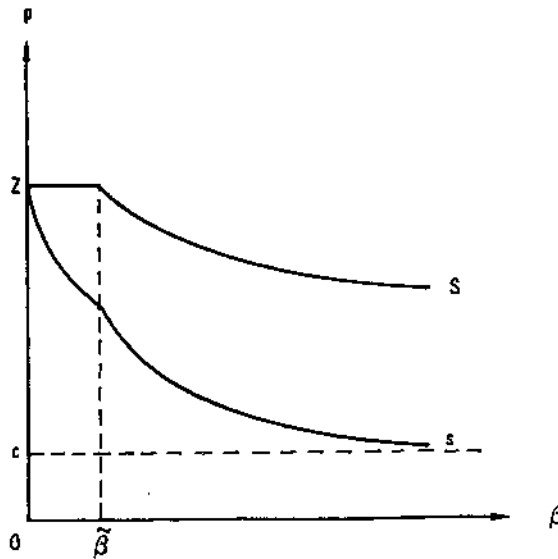


FIGURE 3(a)

The effect of adjustment costs on real prices

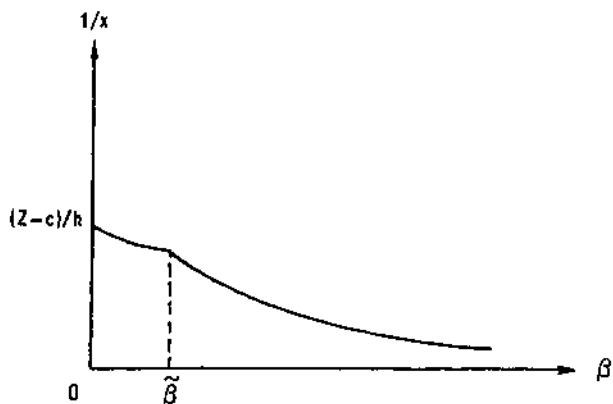


FIGURE 3(b)

The effect of adjustment costs on the density of firms in the market

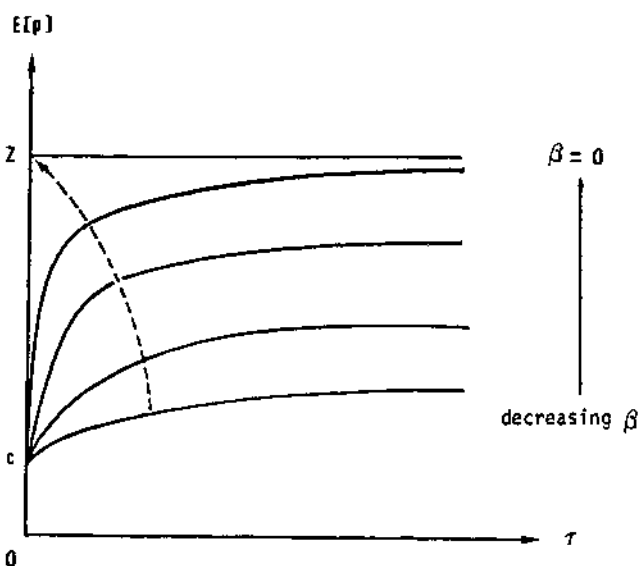


FIGURE 3(c)

The effect of adjustment and search costs on the average price

dispersion and uncertainty (cf. in particular Fischer (1981), (1984) and the references therein). The Sheshinski and Weiss (1977) model features a relationship between inflation and relative price variability at the individual firm level, while Caplin and Spulber (1986) establish a relationship between inflation and relative price dispersion by aggregating a continuum of independent  $(S, s)$  policies. Competition between sellers, however, constrains the amount of price variation—the  $(S, s)$  range—sustainable in the market, and should be explicitly incorporated into the analysis. This is done here through search, and the following proposition establishes a causal relationship between the aggregate rate of inflation and the amount of price dispersion—translating for buyers into price uncertainty—in the market. This relationship is robust to the effects of competition and entry, each of which works in the other direction.

**Proposition 4.** *The equilibrium is a continuous and monotonic function of the inflation rate  $g$ , and there exists  $\tilde{g} \geq 0$  such that search is binding if and only if  $g > \tilde{g}$ ; the equilibrium is differentiable except at  $\tilde{g}$ . As  $g$  decreases to zero, the equilibrium bounds  $(S, s)$  increase to limits  $(\underline{S}, \underline{s})$ , while price dispersion decreases to  $(\underline{S} - \underline{s})/\underline{s} > 0$  and the density of firms increases to a limit of  $1/\underline{x} < (Z - c)/h$ .<sup>16</sup> Moreover, this limiting equilibrium, as a function of  $\tau$ , possesses all the properties listed in Proposition 3.1; in particular, search is binding ( $\underline{S} < Z$  and  $d\underline{S}/d\tau > 0$ ) if and only if:*

$$\tau < \tilde{\tau} \equiv Z \left[ 1 - \frac{1 - \tilde{\sigma}}{\text{Log}(1/\tilde{\sigma})} \right], \quad \text{where } \tilde{\sigma} = \frac{h + r\beta c / Z}{h + r\beta}.$$

*Proof.* See appendix: ||

This result is illustrated in Figures 4(a) and 4(b).<sup>17</sup> Figure 3(c) applies to variations in  $g$  as well as  $\beta$ , except that the limiting curve as  $g$  tends to zero is continuous and strictly below the static one.

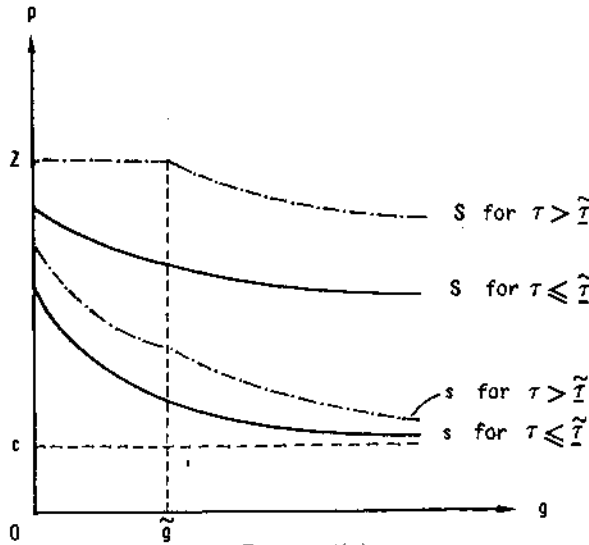


FIGURE 4(a)  
The effect of inflation on real prices

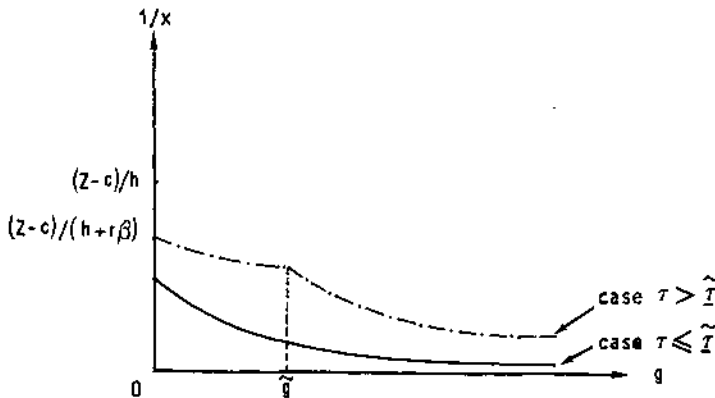


FIGURE 4(b)  
The effect of inflation on the density of firms in the market

In the presence of frictions, inflation is thus far from neutral; not only does it impose costly price adjustments, but also:

(a) It erodes the monopoly power of price setters by driving their real prices away from the profit-maximizing level; this is the usual Sheshinski-Weiss result (working here only in the downward direction because of the reservation-price nature of demand).

(b) It generates *price dispersion*, which makes search potentially profitable (a credible threat) and thereby *increases price competition*, resulting in lower real prices on the market and the exit of some firms.

(c) At the *macroeconomic* level, *inflation alters the relative prices* of the different sectors—here labour and the good—even though these aggregate prices are growing at the same rate (cf. Lemma 1).<sup>18</sup>

Unlike the case of  $\beta$ , the monopoly price result is not even approximated by economies with arbitrarily small but positive inflation rates: as  $g$  converges to zero,  $s$  stays bounded away from  $S$  (cf. Theorem 2.1), so that there remains a finite amount of price dispersion in the market, which generates competition if search costs are lower than some finite limit. Thus it is more the presence of the cost of price adjustment than that of significant inflation which ensures a smooth functioning of the market—the latter essentially amplifies the effect of the former. One could even consider a static model in which there happened to be some price dispersion ( $\sigma < 1$ ); given a cost of price adjustment  $\beta$  such that  $x(S-s)/r < \beta$ , or  $\sigma > 1 - r\beta/Sx$ , and a search cost low enough for search to be binding, this price dispersion would constitute an equilibrium. Of course, the notion of adjustment cost is intrinsically a dynamic one, and the reason for the initial dispersion and its exact form would have to be made explicit, as they are here through inflation.

## 5. DISCUSSION

### 5.1 *Search and price dispersion:*

The most common approach used by previous authors to circumvent the unsatisfactory monopoly price result of the basic model with identical consumers and firms is to allow for consumer heterogeneity. More precisely, it is assumed that buyers have different search costs (an exception is Diamond (1987*b*) where they differ in their valuations of the good). Even in this context, any improvement over the single-price result is conditional upon additional assumptions about either: (a) the existence of “enough” consumers with infinitesimal search cost, in the sense of a mass-point of the distribution at zero (von zur Muehlen (1980)), or a positive density in the neighbourhood of zero (Rob (1985), Stiglitz (1985)); (b) the elasticity of demand or unit costs; (c) the “experimental behaviour” of firms who try to discover their demand curve (Axell (1977)). Reinganum (1979), on the other hand, introduced heterogeneity among firms instead of consumers by assuming different unit costs and hence different monopoly prices. A price dispersion equilibrium is thus generated, in which all firms with cost below some critical level charge their monopoly price, while all others bunch (into an atom of the price distribution) at the monopoly price corresponding to this critical cost. In the terms of this paper, it could be said that search is binding on the more inefficient firms, while only preferences (the elasticity of demand) limit the prices which the more efficient ones can charge.<sup>19</sup> The Reinganum model, however, still falls short of completely reconciling equilibrium price dispersion with search and providing a fully realistic description of monopolistic competition: indeed, for the firms among which price dispersion exists, search has no effect and the monopoly price result remains; conversely, there is no price dispersion among the firms for which search matters. Moreover, each firm is earning a different level of profits.



In the long run, efficient technologies can be replicated, generating entry and exit and shifting the market structure towards the classical limit, where there remain only identical, efficient firms—and thus no price dispersion.

The model presented here is much closer than those mentioned just above to the original paradoxical case, since it involves identical consumers and firms, inelastic demand and constant unit costs. It clearly shows that the heart of the problem is *not heterogeneity* versus homogeneity, but the relative sizes of the *frictions*<sup>20</sup> that affect both sides of the market. Elastic demand, decreasing returns to scale or any arbitrary distribution of search costs could be added, but are not necessary. Similarly, using the results of Sheshinski and Weiss (1982) and Caplin and Spulber (1986), the rate of inflation could be made stochastic without affecting the results.

### 5.2. *Other equilibria*

Condition E1, which restricts attention to a particular class of equilibria (not strategies), could be considered arbitrarily selective. The existence result is not in question, but uniqueness may not hold any more once other types of equilibria are allowed: there might be asymmetric  $(S, s)$  equilibria, non- $(S, s)$  equilibria, etc. Perhaps this plethora is the best justification for focusing on a class which one judges to be both sensible (not to mention tractable), and well suited to the idea of monopolistic competition.

Even in the class of symmetric  $(S, s)$  equilibria, however, there exists another member which was excluded by condition E2: if all firms start at the same price and remain synchronized, there is no price dispersion, no search, and the simultaneous adjustment thus takes place to  $Z$ . This equilibrium replicates the monopoly price result and therefore shares none of the attractive comparative statics and limiting properties of the one selected by E2. It is therefore worth recalling that it was not excluded arbitrarily, but for several important reasons: inconsistency with the assumed aggregate inflationary process, and instability with respect to heterogeneity among firms, arising from slightly different costs, idiosyncratic shocks, or randomisation (cf. Section 1.3). The invariant distribution equilibrium studied here and this synchronized equilibrium in fact exhaust the class of symmetric  $(S, s)$  equilibria: any initial distribution which involves price dispersion will generate search—if search costs are low enough; if it is not invariant over time, search behaviour and therefore demand will not be stationary, destroying the optimality of  $(S, s)$  policies.

### 5.3. *Inflation and welfare*

Because of the simplifying assumption made about consumers' preferences, inflation here is entirely to their advantage and to the detriment of firms. With a more elastic demand curve, the upper real price bound  $S$  would be greater than the static monopoly price, so that inflation's pressure on real prices would not be downward only. Welfare conclusions would then be ambiguous, but the nature and comparative static properties of the equilibrium would most likely remain the same. In a stochastic setting, inflation could also lead to a deterioration in consumers' knowledge of the price distribution, thereby decreasing the profitability of search and reinforcing sellers' monopoly power, as in Gertner's (1987) static model of duopoly with search. Another important concern that one may have over the effect of inflation on welfare is the idea that if increased inflation generates increased price dispersion (as is the case here), it will also cause more resources to be spent on search. In this respect, the present model lacks both realism and predictive

power: search is essentially a credible threat, which consumers faced with the maximum price in the market are just indifferent between carrying out and not carrying out. In Bénabou (1986), the model is therefore generalized to heterogeneous consumers, generating active search in equilibrium; these results are extended in Bénabou (1987b) and used to analyse the relationships between inflation, the resources spent on search, and welfare.

CONCLUSION

The  $(S, s)$  pricing model has been extended to monopolistic competition by endogenously determining firms' demand curve from consumer search. The resulting equilibrium embodies optimal price-setting, sequential search and entry, and features a causal relationship between anticipated inflation and price dispersion. It also leads to the conclusion that Bertrand competition and the monopoly price result of Diamond (1971) are polar limiting cases of a smoother, monopolistically competitive equilibrium with frictions (search and price adjustment costs) on both sides of the market. It should still be possible, however, to incorporate additional and more complex features into the model, so as to further increase its realism and macroeconomic relevance.

APPENDIX

*Proof of Lemma 2.1.* Since  $d\mu(p) = dp/[p \text{Log}(S/s)]$ :

$$E[p] = \left( \int_s^S p \cdot dp/p \right) / (\text{Log}(S/s) = (S-s)/\text{Log}(S/s));$$

$$E[p^2] = \left( \int_s^S p^2 \cdot dp/p \right) / (\text{Log}(S/s) = (S^2-s^2)/(2 \text{Log}(S/s)));$$

thus

$$\begin{aligned} \text{Var}[p]/E[p]^2 &= [(S+s)/(S-s)](\text{Log}(S/s)/2) - 1 \\ &= \text{Log}(1/\sigma)[(1/\sigma+1)/(1/\sigma-1)]/2 - 1 = h(1/\sigma) \end{aligned}$$

where, for all  $y > 1$ :  $y(y-1)^2 h'(y) = y^2 - 1 - 2y \text{Log}(y) \equiv m(y)$ , with:  $m'(y) = 2(y-1 - \text{Log}(y)) > 0$  and  $m(1) = 0$ ; thus  $m(y) > 0$ , or  $h'(y) > 0$ , for all  $y > 1$ . ||

*Proof of Theorem 2.1.* The proof of the optimality of a recursive—i.e.  $(S, s)$ —real price policy given in the appendix of Sheshinski and Weiss (1977) relies only on the stationarity—and not on the functional form—of the demand curve. Their characterisation of the optimal  $(S, s)$  through first-order conditions, however, no longer holds here because the reservation-price nature of demand imposes a boundary constraint  $S \leq Q$  on the optimization problem. This results in a non-interior solution for  $S$ .

Consider a feasible  $(S, s)$  rule where  $S < Q$ , and let  $T = \text{Log}(S/s)/g$ . Real intertemporal profits (gross of the fixed cost  $h/r$ ) are:

$$V = -\beta + \int_0^T x(Se^{-rt} - c)e^{-rt} dt + Ve^{-rT}.$$

Now let the firm deviate at some time when a price change is called for, adjusting to  $Q$  instead of  $S$ , then readjusting to  $S$  after a time  $T$  and resuming  $(S, s)$  from there on. This perturbation yields:

$$W = -\beta + \int_0^T x(Qe^{-rt} - c)e^{-rt} dt + Ve^{-rT} > V$$

because  $Q > S$ . Therefore, a policy with  $S < Q$  cannot be optimal.

Similarly, consider an  $(S, s)$  rule where  $S > Q$ . After an adjustment to  $S$ , sales are zero during a time  $\delta = \text{Log}(S/Q)/g > 0$ ; note that  $\delta < T = \text{Log}(S/s)/g$  if any sales are to be made. Moreover:

$$V = -\beta + \int_\delta^T x(Qe^{-r(t-\delta)} - c)e^{-rt} dt + Ve^{-rT}$$

Now let the firm deviate at some time when a price change is called for, adjusting to  $Q$  instead of  $S$ , then resuming the same  $(S, s)$  policy (the next adjustment thus takes place at  $s$ , after a time  $T - \delta$ ); this yields:

$$\begin{aligned} W &= -\beta + \int_0^{T-\delta} x(Qe^{-rt} - c)e^{-rt} dt + Ve^{-r(T-\delta)} \\ &= -\beta + \left[ \int_0^T x(Qe^{-r(t-\delta)} - c)e^{-rt} dt + Ve^{-rT} \right] e^{r\delta} \end{aligned}$$

so that  $W + \beta = e^{r\delta}(V + \beta)$ , hence  $W > V$ , and  $S > Q$  cannot be optimal.

Given that the optimal  $S$  is  $Q$ , profits (gross of fixed costs  $h/r$ ) as a function  $V_T$  of  $T = \text{Log}(Q/s)/g$  are:

$$V_T = \left[ \int_0^T x(Qe^{-rt} - c)e^{-rt} dt - \beta \right] / (1 - e^{-rT})$$

or

$$V_T = -cx/r + Qx[1 - \beta(r+g)/Qx - e^{-(r+g)T}] / [(r+g)(1 - e^{-rT})]. \quad (\text{A1})$$

Alternatively, as a function of  $\sigma$  (let  $u = e^{-rt}$ ):

$$V(\sigma) = -cx/r + Qx \left[ \int_\sigma^1 u^{r/g} du - \beta g/Qx \right] / \left[ g(1 - \sigma^{r/g}) \right]. \quad (\text{A2})$$

Hence

$$V'(\sigma)(1 - \sigma^{r/g})g = -\sigma^{-1+r/g}x[\sigma Q - r(V(\sigma)/x + c/r)],$$

so that

$$\text{sgn}\{V'(\sigma)\} = \text{sgn}\{rV(\sigma) - x(s - c)\} \quad (\text{A3})$$

where  $\text{sgn}\{z\}$  denotes the sign of  $z$ . Thus  $V'(\sigma) \geq 0$  if and only if:

$$\begin{aligned} \sigma &\leq (rV(\sigma)/x + c)/Q = r \left[ \int_\sigma^1 u^{r/g} du - \beta g/Qx \right] / \left[ g(1 - \sigma^{r/g}) \right] \\ &= r[1 - (r+g)\beta/Qx - \sigma^{1+r/g}] / [(r+g)(1 - \sigma^{r/g})] \end{aligned}$$

or

$$f(\sigma) \equiv (r+g)\sigma - g\sigma^{1+r/g} - r[1 - (r+g)\beta/Qx] \geq 0 \quad (\text{A4})$$

For all  $\sigma \in (0, 1)$ :  $f'(\sigma) = (r+g)(1 - \sigma^{r/g}) > 0$ ,  $f(0) = -r[1 - (r+g)\beta/Qx] < 0$  by hypothesis, and  $f(1) = r(r+g)\beta/Qx > 0$ . Therefore  $f$  has a unique zero—i.e. by (A3)  $V(\sigma)$  has a unique global maximum— $\sigma \equiv \sigma^*(r, g, \beta/Qx) \in (0, 1)$ . Equivalently,  $V_T$  has a unique maximum  $T^*(r, g, \beta/Qx) \in (0, +\infty)$ , with  $\sigma^* = e^{-sT^*}$ . Moreover, by (A3),  $s = \sigma^*Q$  satisfies:  $x(s - c) = rV_T$ .

Let us now turn to the comparative statics of  $\sigma$  with respect to  $\beta$  and  $g$ . Since  $f$  is parametrized by  $r, g$ , and  $\beta' \equiv \beta/Qx$ , and continuously differentiable in these parameters, so is  $\sigma = \sigma^*(r, g, \beta')$ , and:

$$\partial\sigma/\partial\beta' = -(\partial f/\partial\beta')/(\partial f/\partial\sigma) = -r/(1 - \sigma^{r/g}) < 0;$$

$$\partial\sigma/\partial g = -(\partial f/\partial g)/(\partial f/\partial\sigma) < 0, \quad \text{because } \partial f/\partial\sigma > 0$$

and

$$\partial f/\partial g = \sigma[1 - \sigma^{r/g}(1 - (r/g)\text{Log}(\sigma))] + r\beta/Qx > 0$$

since

$$\text{Log}(1/\sigma^{r/g}) < 1/\sigma^{r/g} - 1.$$

Thus  $\sigma$  is decreasing in both  $\beta'$  and  $g$ , and therefore has finite limits  $\sigma'$  and  $\sigma''$  as these parameters go to zero. Taking limits in:  $f(\sigma; r, g, \beta') = 0$  as  $\beta' = \beta/Qx$  tends to zero yields:  $(r+g)\sigma' - g(\sigma')^{1+r/g} = r$ , to which the only solution is  $\sigma' = 1$ . Similarly, taking limits as  $g$  tends to zero yields:  $r\sigma'' = r(1 - r\beta/Qx)$ , hence a limit  $\sigma'' = 1 - r\beta/Qx$ . ||

*Proof of Lemma 2.2.* By definition (equations (7) and (9) in the text):

$$\Phi(\sigma, x; r, g, \beta, h) = (r+g)\sigma - g\sigma^{1+r/g} - r[1 - (r+g)\beta\sigma/(cx + h)] \quad (\text{A5})$$

$$\Psi(\sigma, x; \tau, h) = (1 - \sigma)/\text{Log}(1/\sigma) + \tau\sigma/(c + h/x) - 1 \quad (\text{A6})$$

for all  $\sigma \in (0, 1)$  and  $x \in (0, +\infty)$ . Hence:

$$\begin{aligned}\partial\Phi(\sigma, x)/\partial\sigma &= (r+g)(1-\sigma^{r/g}) + r(r+g)\beta/(cx+h) > 0; \\ \partial\Phi(\sigma, x)/\partial x &= -r(r+g)\beta\sigma c/(cx+h)^2 < 0; \\ \Phi(0, x) &= -r < 0; \Phi(1, x) = r(r+g)\beta/(cx+h) > 0;\end{aligned}$$

and

$$\begin{aligned}\partial\Psi(\sigma, x)/\partial\sigma &= [1/\sigma - 1 - \text{Log}(1/\sigma)]/\text{Log}(\sigma)^2 + \tau(c+h/x) > 0; \\ \partial\Psi(\sigma, x)/\partial x &= \tau\sigma h/(cx+h)^2 > 0; \\ \Psi(0, x) &= -1 < 0; \Psi(1, x) = \tau/(c+h/x) > 0.\end{aligned}$$

The rest of the lemma follows immediately from the Implicit Function Theorem. Moreover,  $\sigma'_0(x) > 0 > \sigma'_1(x)$  and the limiting equalities:  $\Phi(1, +\infty) = 0 = \Psi(1, 0) = 0$  imply the following useful result:

$$\sigma_0(0) < \sigma_0(+\infty) = 1 = \sigma_1(0) > \sigma_1(+\infty). \quad \parallel \quad (\text{A7})$$

*Proof of Theorem 2.2.* An equilibrium is a solution to (equation (12) in the text):

$$\sigma_0(x) = \max\{\sigma_1(x), \sigma_2(x)\} \equiv \sigma_3(x). \quad (\text{A8})$$

By Lemma 2.2,  $\sigma_0 - \sigma_3$  is increasing on  $(0, +\infty)$ . Moreover, by (A7):  $\sigma_0(0) < \sigma_3(0)$  and  $\sigma_0(+\infty) = 1 > \max\{\sigma_1(+\infty), c/Z\} = \sigma_3(+\infty)$ . There exists therefore a *unique solution*  $x^*$  to (A8), defining the equilibrium:

$$\begin{aligned}(x^*, \sigma^* = \sigma_0(x^*) = \sigma_3(x^*), s^* = c + h/x^*, \\ S^* = \min\{Z, (c + h/x^*)/\sigma_1(x^*)\} = \min\{Z, \tau\theta(r, g, \beta/S^*x^*)\})\end{aligned} \quad (\text{A9})$$

by equation (4c) in the text.

It now remains to examine the values of  $\tau$  for which search is binding in equilibrium. The function  $\sigma_0 - \sigma_2$  is increasing on  $(0, +\infty)$ , with limiting values of  $-\infty$  at  $x=0^+$  and  $1 - c/Z$  at  $x = +\infty$  (by (A7)). Therefore:

$$(\exists! \bar{x} > 0) \quad (\sigma_0(\bar{x}) = \sigma_2(\bar{x})). \quad (\text{A10})$$

Consider now, for any  $x$ , the equation defining  $\sigma_1(x)$ :

$$\Psi(\sigma, x; \tau, h) = (1-\sigma)/\text{Log}(1/\sigma) + \tau\sigma/(c+h/x) - 1 = 0. \quad (\text{A11})$$

For a given  $x$ , it can be viewed as an implicit equation in  $\tau$  and  $\sigma$ ; the solution  $\sigma_1(x)$  is therefore also a function of  $\tau$ , which will be made clear by writing it as  $\sigma_1(x, \tau)$ . Moreover:

$$\begin{aligned}\partial\sigma_1(x, \tau)/\partial\tau &= -(\partial\Psi(\sigma_1, x; \tau)/\partial\tau)/(\partial\Psi(\sigma_1, x; \tau)/\partial\sigma); \\ \partial\Psi(\sigma_1, x; \tau)/\partial\sigma &> 0 \quad \text{by Lemma II.2;} \\ \partial\Psi(\sigma_1, x; \tau)/\partial\tau &= \sigma_1/(c+h/x) > 0; \quad \text{therefore} \\ \partial\sigma_1(x, \tau)/\partial\tau &< 0.\end{aligned} \quad (\text{A12})$$

It is clear from (A11) that  $\sigma_1(x, 0) = 1$  for any  $x > 0$ , while as  $\tau$  tends to  $+\infty$ ,  $\sigma_1(x, \tau) \approx (c+h/x)/\tau \rightarrow 0$ . Taking in particular  $x = \bar{x}$ :  $\sigma_1(\bar{x}, \cdot)$  decreases from 1 to 0 on  $[0, +\infty)$ , while  $\sigma_0(\bar{x}) \in (0, 1)$ . Hence:

$$(\exists! \bar{\tau} > 0) \quad (\sigma_1(\bar{x}, \bar{\tau}) = \sigma_0(\bar{x}) = \sigma_2(\bar{x})). \quad (\text{A13})$$

It will now be shown that search is binding (i.e.  $\sigma_1(x^*, \tau) > \sigma_2(x^*)$ ) if and only if  $\tau < \bar{\tau}$ . By definition of  $\bar{\tau}$ :

(a) For  $\tau \geq \bar{\tau}$ :  $\sigma_1(\bar{x}, \tau) \leq \sigma_2(\bar{x})$  hence:  $\sigma_0(\bar{x}) = \sigma_3(\bar{x}, \tau) = \sigma_2(\bar{x})$  by (A8) and (A10); thus  $\bar{x}$  is still the equilibrium, and  $S$  is equal to  $(c+h/\bar{x})/\sigma_2(\bar{x}) = Z$  (preferences binding).

(b) For  $\tau < \bar{\tau}$ :  $\sigma_1(\bar{x}, \tau) > \sigma_2(\bar{x})$  hence:  $\sigma_0(\bar{x}) = \sigma_2(\bar{x}) < \sigma_3(\bar{x}, \tau)$ . Since  $\sigma_3 - \sigma_0$  is decreasing, its zero—the equilibrium—must occur at some:

$$x^* > \bar{x}, \quad \sigma^* = \sigma_0(x^*) > \bar{\sigma}. \quad (\text{A14})$$

This implies, by (A10):  $\sigma_2(x^*) < \sigma_0(x^*) = \sigma_3(x^*, \tau)$ , which in turn requires:  $\sigma_3(x^*, \tau) = \sigma_1(x^*) > \sigma_2(x^*)$  (search binding).  $\parallel$

*Proof of Proposition 3.1.* By Theorem 2.2, when  $\tau \geq \bar{\tau}$ , preferences are binding:

$$S = Z, \quad x = \bar{x}, \quad \sigma = \sigma_0(\bar{x}) = \sigma_2(\bar{x}), \quad s = c + h/\bar{x} \quad (\text{A15})$$

so that the equilibrium is independent of  $\tau$ . When  $\tau < \hat{\tau}$ , search is binding and:  $\sigma = \sigma_0(x) = \sigma_1(x, \tau)$ ; differentiation yields:

$$[\sigma'_0(x) - \partial\sigma_1(x, \tau)/\partial x](dx/d\tau) = \partial\sigma_1(x, \tau)/\partial\tau < 0$$

by (A12). Hence, since  $\sigma'_0(x) > 0 > \partial\sigma_1(x, \tau)/\partial x$  by Lemma 2.2:

$$dx/d\tau < 0. \tag{A16}$$

Then:  $\sigma = \sigma_0(x)$ ,  $s = c + h/x$  and  $S = s/\sigma$  imply:

$$d\sigma/d\tau < 0, \quad ds/d\tau > 0, \quad dS/d\tau > 0. \tag{A17}$$

As  $\tau$  goes to zero,  $x$  and  $\sigma$  increase to limits  $\hat{x}$  and  $\hat{\sigma}$ . Taking limits in (A11) (binding search condition) yields:  $(1 - \hat{\sigma})/\text{Log}(1/\hat{\sigma}) = 1$ , or  $\hat{\sigma} = 1$ . Taking now limits in  $\Phi(\sigma, x) = 0$  (cf. (A5)) yields:  $\hat{x} = +\infty$ , implying finally that both  $s = c + h/x$  and  $S = s/\sigma$  converge to  $c$ .  $\parallel$

*Proof of Proposition 3.2. Monotonicity.* The direct dependence of  $\sigma_0$  on  $\beta$  (in addition to the indirect one through  $x$ ) will be made clear by denoting it as  $\sigma_0(x, \beta)$ . To examine how the equilibrium varies with  $\beta$ , we will first focus on the threshold levels  $\hat{x}$  and  $\hat{\tau}$ . Denoting as  $\hat{\sigma}$  the quantity  $\sigma_2(\hat{x}) = \sigma_0(\hat{x}, \beta)$ , (cf. (A10)), and differentiating with respect to  $\beta$ :

$$[\partial\sigma_0(\hat{x}, \beta)/\partial x - \sigma'_2(\hat{x})](d\hat{x}/d\beta) = -\partial\sigma_0(\hat{x}, \beta)/\partial\beta.$$

But since

$$\begin{aligned} \partial\sigma_0(\hat{x}, \beta)/\partial\beta &= -(\partial\Phi(\hat{\sigma}, \hat{x}; \beta)/\partial\beta)/(\partial\Phi(\hat{\sigma}, \hat{x}; \beta)/\partial\sigma) \\ &= -r(r + g)\hat{\sigma}/[(c\hat{x} + h)(\partial\Phi(\hat{\sigma}, \hat{x}; \beta)/\partial\sigma)] < 0, \end{aligned} \tag{A18}$$

$$d\hat{x}/d\beta > 0. \tag{A19}$$

Similarly,  $\hat{\tau}$  is defined in (A13) by:  $\sigma_1(\hat{x}, \hat{\tau}) = \hat{\sigma} = \sigma_2(\hat{x})$ , so

$$[\partial\sigma_1(\hat{x}, \hat{\tau})/\partial x - \sigma'_2(\hat{x})](d\hat{\tau}/d\beta) = -(\partial\sigma_1(\hat{x}, \hat{\tau})/\partial\tau)(d\hat{\tau}/d\beta).$$

Therefore, by (A12) and (A19),  $d\hat{\tau}/d\beta$  has the sign of  $\partial\sigma_1(\hat{x}, \hat{\tau})/\partial x - \sigma'_2(\hat{x})$ , which is that of:

$$-\partial\Psi(\hat{\sigma}, \hat{x}; \hat{\tau})/\partial x - \sigma'_2(\hat{x})\partial\Psi(\hat{\sigma}, \hat{x}; \hat{\tau})/\partial\sigma = (h/Z\hat{x}^2)[k'(\hat{\sigma}) + \hat{\tau}\hat{x}/(c\hat{x} + h)] - \hat{\tau}h\hat{\sigma}/(c\hat{x} + h)^2$$

where for all  $\sigma \in (0, 1)$ , one defines:

$$k(\sigma) = (1 - \sigma)/\text{Log}(1/\sigma). \tag{A20}$$

$$k'(\sigma) = [1/\sigma - 1 - \text{Log}(1/\sigma)]/[\text{Log}(1/\sigma)^2] > 0. \tag{A21}$$

Moreover, leaving out the term in  $k'(\hat{\sigma})$  in the above expression yields:

$$\hat{\tau}h/[(c\hat{x} + h)Z\hat{x}] - \hat{\tau}h\hat{\sigma}/(c\hat{x} + h)^2,$$

which is proportional to

$$c\hat{x} + h - Z\hat{x}\hat{\sigma} = h - \hat{x}(Z\hat{\sigma} - c) = h - \hat{x}(Z\sigma_2(\hat{x}) - c) = 0$$

by the definitions of  $\hat{\sigma}$  and  $\sigma_2(\cdot)$ . Therefore:

$$\partial\sigma_1(\hat{x}, \hat{\tau})/\partial x - \sigma'_2(\hat{x}) > 0 \tag{A22}$$

$$d\hat{\tau}/d\beta > 0. \tag{A23}$$

Consider now any  $\beta$  and  $\beta'$ , with  $\beta > \beta' > 0$ . Equilibrium values corresponding to  $\beta'$  will be denoted by a prime. By (A23),  $\hat{\tau} > \hat{\tau}'$ , so three cases arise:

(a) For  $\tau \cong \hat{\tau} > \hat{\tau}'$ , preferences are binding in both the equilibrium with search cost  $\tau$  and adjustment cost  $\beta$ , and in that with  $\tau$  and  $\beta'$ . Thus:  $x = \hat{x} > \hat{x}'$  by (A19);  $\sigma = \sigma_2(\hat{x}) < \sigma_2(\hat{x}') = \sigma'$ ;

$$s = c + h/\hat{x} < c + h/\hat{x}' = s', \quad S = Z = S'.$$

(b) For  $\hat{\tau} > \tau \cong \hat{\tau}'$ , preferences are binding with  $\beta'$  and search with  $\beta$ , so:  $x > \hat{x} > \hat{x}' = x'$  by (A14) and (A19);  $\sigma = \sigma_1(x) < \sigma_1(x') < \sigma_2(x') = \sigma'$ ;

$$s = c + h/x < c + h/\hat{x}' = s'; \quad S < Z = S'.$$

(c) For  $\hat{\tau} > \hat{\tau}' > \tau$ , search is binding in both cases; this is also true for any  $\beta'' \in (\beta', \beta)$ , because  $\tau < \hat{\tau}' < \hat{\tau}''$ . Thus, for any such  $\beta''$ :  $\sigma_0(x'', \beta'') = \sigma_1(x'')$ , and differentiation yields:

$$[\partial\sigma_0(x'', \beta'')/\partial x - \sigma'_1(x'')](dx''/d\beta) = -\partial\sigma_0(x'', \beta'')/\partial\beta > 0$$

by (A18). This in turn implies:  $dx''/d\beta > 0$  for all  $\beta'' \in (\beta', \beta)$  or:

$$dx/d\beta > 0 \tag{A24}$$

$$ds/d\beta < 0, \quad d\sigma/d\beta < 0, \quad d\theta/d\beta < 0; \quad dS/d\beta = d(\tau\theta)/d\beta < 0 \tag{A25}$$

on the interval  $(\beta', \beta)$ . Finally, these inequalities imply:

$$S < S', \quad s < s', \quad \sigma < \sigma', \quad x > x'. \tag{A26}$$

which concludes the proof of the equilibrium's monotonicity in  $\beta$ .

*Limits:* As  $\beta$  tends to zero, the monotone functions  $\tilde{x}, \tilde{\sigma}, \tilde{\tau}, S, s, \sigma$  and  $x$  possess limits  $\tilde{x}, \tilde{\sigma}, \tilde{\tau}, \tilde{S}, \tilde{s}, \tilde{\sigma}$  and  $\tilde{x}$ . Since  $\tilde{\sigma} = \sigma_0(\tilde{x}, \beta)$ :  $\Phi(\tilde{\sigma}, \tilde{x}) = 0$ ; taking limits as  $\beta$  goes to zero yields (cf. (A5)):  $(r + g)\tilde{\sigma} - g\tilde{\sigma}^{1+r/g} - r = 0$ , so  $\tilde{\sigma} = 1$ . But  $z\tilde{\sigma} = z\sigma_2(\tilde{x}) = c + h/\tilde{x}$ ; hence:  $\tilde{x} = h/(Z - c)$ . Similarly,  $\tilde{\sigma} = \sigma_1(\tilde{x}, \beta; \tilde{\tau})$ , so  $\Psi(\tilde{\sigma}, \tilde{x}; \tilde{\tau}) = 0$ ; taking limits as  $\beta$  tends to zero yields (cf. (A6)):  $1 + \tilde{\tau}/Z - 1 = 0$ , so that:

$$\tilde{\sigma} = 1, \quad \tilde{x} = h/(Z - c), \quad \tilde{\tau} = 0. \tag{A27}$$

Thus, for any  $\tau > 0$ :  $\tilde{\beta} = \min\{\beta > 0 | \tilde{\tau} \geq \tau\} > 0$  is well defined and positive, and  $\tau > \tilde{\tau}$  for  $\beta < \tilde{\beta}$ . Preferences are then binding:  $S = Z$ ,  $x = \tilde{x}$ ,  $\sigma = \sigma_2(\tilde{x})$ ,  $s = Z\sigma_2(\tilde{x})$ . Taking limits as  $\beta$  goes to zero:

$$S = Z, \quad x = h/(Z - c), \quad \sigma = 1, \quad s = Z \quad || \tag{A28}$$

*Proof of Proposition 4. Monotonicity.* As was done with  $\beta$ ,  $\sigma_0$  will be denoted here as  $\sigma_0(x, g)$ . First for all positive  $g$  and  $x$ :

$$\partial\sigma_0(x, g)/\partial g < 0 \tag{A29}$$

because it has the opposite sign of:

$$\partial\Phi(\sigma_0, x; g)/\partial g = (\sigma_0)^{1+r/g} \{[(\sigma_0)^{-r/g} - 1 + (r/g) \text{Log}(\sigma_0)] + r\beta\sigma_0/(cx + h)\} > 0.$$

Replacing  $\beta$  by  $g$  in the monotonicity proofs of Proposition 3.2 yields by exactly the same calculations:

$$d\tilde{x}/dg > 0, \quad d\tilde{\tau}/dg > 0 \tag{A30}$$

$$dx/dg > 0, \quad d\sigma/dg < 0, \quad ds/dg < 0, \quad dS/dg \leq 0, \tag{A31}$$

with the last inequality being strict (search binding) when  $\tau < \tilde{\tau}$ , which by (A30) is equivalent to:  $g > \tilde{g} = \inf\{g > 0 | \tilde{\tau} \geq \tau\} \geq 0$ .

*Limits.* The limiting equilibrium  $(\tilde{x}, \tilde{\sigma}, \tilde{\tau}; \tilde{x}, \tilde{\sigma}, \tilde{s}, \tilde{S})$  as  $g$  tends to zero is different from that obtained as  $\beta$  tends to zero. Taking limits in  $\Phi(\tilde{\sigma}, \tilde{x}; g) = 0$  (because  $\tilde{\sigma} = \sigma_0(\tilde{x}, g)$ ) yields (cf. (A5)):

$$\tilde{\sigma} + r\beta\tilde{\sigma}/(c\tilde{x} + h) - 1 = 0.$$

But:

$$\tilde{\sigma} = \sigma_2(\tilde{x}) = (c + h/\tilde{x})/Z;$$

finally, eliminating  $\tilde{\sigma}$ :

$$\tilde{x} = (h + r\beta)/(Z - c), \quad \tilde{\sigma} = (h + r\beta c/Z)/(h + r\beta). \tag{A32}$$

Similarly,  $\tilde{\sigma} = \sigma_1(\tilde{x}, g; \tilde{\tau})$  or  $\Psi(\tilde{\sigma}, \tilde{x}; \tilde{\tau}, g) = 0$ ; taking limits (cf. (A6)) leads to:  $\tilde{\tau}\tilde{\sigma}/(c + h/\tilde{x}) = 1 - (1 - \tilde{\sigma})/\text{Log}(1/\tilde{\sigma})$  or:

$$\tilde{\tau} = Z[1 - (1 - \tilde{\sigma})/\text{Log}(1/\tilde{\sigma})] > 0 \tag{A33}$$

because  $\tilde{\sigma} \in (0, 1)$  by (A32). Formulas (A32) and (A33) should be contrasted to (A27). It will now be shown that the limiting equilibrium has search binding when  $\tau < \tilde{\tau}$ .

(a) For  $\tau > \tilde{\tau}$ : when  $g$  is small enough,  $\tau > \tilde{\tau}$  by (A30), and preferences are binding:  $S = Z$ ,  $x = \tilde{x}$ ,  $\sigma = \sigma_2(\tilde{x})$ ,  $s = Z\sigma = Z\sigma_2(\tilde{x})$ . Taking limits as  $g$  tends to zero yields:  $S = Z$ ,  $x = \tilde{x} > h/(Z - c)$ ,  $\sigma = \tilde{\sigma} < 1$  and  $s = Z\tilde{\sigma} < Z$ .

(b) For  $\tau \leq \tilde{\tau}$ : for all  $g > 0$ ,  $\tau < \tilde{\tau}$  by (A30), and search is binding:  $S = \tau\theta < Z$ , and  $x > \tilde{x}$  (by (A14)). Taking now limits as  $g$  goes to zero in  $\Psi(\sigma_1(x, g), x; \tau, g) = 0$  yields:  $k(\tilde{\sigma}) + \tau\tilde{\sigma}/(c + h/\tilde{x}) = 1$ ; since  $\tilde{x} \geq \tilde{x} > 0$  by (A32), this requires  $\tilde{\sigma} < 1$ . Hence:

$$x \geq \tilde{x} > h/(Z - c); \quad \sigma < 1; \quad s = \tilde{S}\tilde{\sigma} \leq Z\tilde{\sigma} < Z. \tag{A34}$$

$$S = \tau/[1 - (1 - \tilde{\sigma})/\text{Log}(1/\tilde{\sigma})] \leq Z. \tag{A35}$$

It only remains to show that (A35) holds with strict inequality, and implies that  $dS/dg < 0$ , when  $\tau < \bar{f}$  (note that  $\sigma$  depends on  $\tau$ , hence (A35) is implicit in this parameter), i.e. that  $S$  possesses the same properties that were established for any  $S$  corresponding to  $g > 0$  in Proposition 3.1. Since that proof relied only on the fact that:

$$\partial\sigma_0(x, g)/\partial x > 0 > \partial\sigma_1(x, \tau)/\partial x \quad \text{and} \quad \partial\sigma_1(x, \tau)/\partial\tau < 0$$

and since the function  $\sigma_1$  is independent of  $g$ , it suffices to show that  $\sigma_0(x, 0)$  is strictly increasing in  $x$ . But (cf. proof of Lemma 2.2):

$$\partial\sigma_0(x, 0)/\partial x = r\beta\sigma_0c/[(cx + h)(cx + h + r\beta)] > 0. \quad \parallel$$

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#### NOTES

1. A very notable exception is the literature on durable goods: Stokey (1981), Bulow (1982), Sobel (1984), Conlisk, Gestner and Sobel (1984), Gul, Sonnenschein and Wilson (1985), etc.

2. The proofs (cf. appendix) will actually be carried out with  $h > 0$  for convenience; the case where  $h = 0$  is obtained as a limit. Note that entry is treated here in a long-term, or comparative statics sense, since its dynamic process is left unspecified.

3. All the paper's results would remain essentially unchanged if the number of firms in the market was fixed instead of endogenous (cf. Bénabou (1986)).

4. In Bénabou (1986) the model is also solved with different specifications of the price adjustment cost, which is allowed to depend on the number of customers or on the firm's new price (this last case corresponds to sales lost because of resources—i.e. labour—diverted to changing prices). All results are robust to the choice of specification.

5. This assumption is convenient but inessential; the results remain unchanged when each search requires a finite length of time, provided it is assumed that consumers have no discount rate and, most importantly, no possibility of recalling past offers.

6. Because search is instantaneous, time is no limit on the number of searches; footnote (10) below will make clear that neither is wealth.

7. This standard assumption ensures that no consumer is kept out of the market because the surplus he can expect from optimal search is smaller than the cost of the first sampling.

8. Caplin and Spulber (1986) deal more generally with any continuous and monotonic inflationary process. For the subcase of constant inflation, see also Rotemberg (1983). Note that among the indexes covered by the Lemma are the arithmetic and geometric averages, and the Divisia index.

9. The invariant distribution in these cases is not exactly log-uniform any more, but what really matters for the problem at hand are its invariance and non-degeneracy, not its precise functional form.

10. Lippman and McCall (1976) show that, with recall allowed, (the same)  $R$  characterizes the optimal decision rule:

(a) in the infinite horizon problem where the constraint imposed by finite wealth (here, labour) on the number of searches is disregarded.

(b) until the last period of the finite horizon problem where this constraint (here  $N \leq (L - Z)/\tau$ ) is taken into account.

Following most of the literature, the paper will implicitly deal with case (a); the model and results derived here, however, are fully robust to the incorporation of the wealth constraint, because (as will be seen below) in equilibrium the maximum real price in the market  $S$  is no greater than  $R$ . Thus: (1) No search actually takes place (hence the constraint is not binding) in equilibrium; (2) In game-theoretic terms, consumers' threat of rejecting, and not coming back to, any price greater than  $R$ , which sustains the equilibrium, is credible even off the equilibrium path. Indeed: (i) they can search at least once, since  $\tau < L - Z$ ; by (b) above, it is thus optimal to reject  $p > R$ ; (ii) since  $S \leq R$ , they immediately find another price  $p' < R$ , hence never come back to  $p$ .

11. This optimisation problem differs from the one covered by Sheshinski and Weiss (1977) because the demand curve is rectangular, so that the optimal  $S$  is not an interior but a corner solution and because they do not explicitly impose that profits be non-negative.

12. This externality is robust to the specification of an adjustment cost  $\beta(x)$  increasing in the number of customers, as long as there are increasing returns to scale in this technology ( $\beta(x)/x$  is decreasing); cf. Bénabou (1986). Another such externality would be for search costs to vary with  $x$ : more firms make search easier, as in Diamond (1982). Since the first effect is already embodied in the model while the second one involves additional assumptions about the search technology, attention is restricted here to the former.

13. Indeed, if all firms do so, search is pointless, so there is no competition and no firm has any incentive to change its price; conversely, in an equilibrium no firm can charge a price above consumers' common reservation price, or else it gets zero demand; thus there is no search either and all firms must be charging the monopoly price.

14. The curves are drawn for  $\tau < \bar{\tau}_\infty = Z[(1 - (1 - c)/Z)/\text{Log}(Z/c)]$ , which ensures that  $\hat{\beta} < +\infty$  (search binding for  $\beta = +\infty$ ). When  $\tau \geq \bar{\tau}_\infty$ :  $S = Z$  for all  $\beta$ . The decrease in the slopes of  $s$  and  $1/x$  at  $\hat{\beta}$  results from equation (A22).

15. Most of Diamond's (1971) paper involves a fixed number of firms; cf. footnote 3 above.

16. As in Sheshinski and Weiss (1977), the sign of the derivative of the periodicity of adjustment  $T^*$  with respect to  $g$  is generally ambiguous.

17. The same remarks as in footnote (14) apply, replacing  $\hat{\beta}$  by  $\hat{g}$  (in the case  $\tau > \bar{\tau}$ ).

18. Section 5.3 discusses an extension of the model (Bénabou (1986), (1987b)) in which inflation generates active search and therefore has an additional resource cost.

19. I am grateful to Peter Diamond for pointing out this analogy. One can also be drawn with Diamond (1987b), where firms which specialize in serving high valuation consumers are constrained by the threat of search, which prevents them from charging these customers their full valuation; firms specializing in low valuation consumers, on the other hand, are constrained only by the latter's preferences (valuations).

20. Another type of friction or market imperfection is the fixed cost  $h$ ; comparative statics and dynamics exercises similar to those presented here for  $\beta$  and  $g$  can be performed with respect to  $h$  (cf. Bénabou (1986)).

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