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Inflation and Efficiency in Search Markets

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This paper examines how inflation affects efficiency and output in monopolistically competitive search markets. It formalizes the conventional wisdom linking higher inflation, price dispersion, and increased resources devoted to search. It also brings to light the induced exit of firms, which reduces rent dissipation. But the most important effect of inflation is how it alters the distribution of real transactions prices. Whether this reduces or promotes efficiency and output is shown to depend critically on preferences and market structure, and especially on whether search costs are large or small relative to consumer surplus.

1. INTRODUCTION

"Because higher inflation tends to be associated with greater dispersion of prices, households and businesses will devote more resources to searching for the lowest price when inflation is high.... Although this activity is productive from the point of view of the individual, from society's point of view it represents a waste because the resources are not being used to produce real goods and services... For (this) reason, resources will not be allocated efficiently."

Economic Report of the President (1990)[1]

This policy concern reflects what Fischer (1984) identified as "a persistent theme in the inflation literature", namely that inflation distorts the allocative role of the price system. Prominent in the standard list of potential inefficiencies is the connection described above inflation, increased price dispersion, and the resources spent on search.

There is a fair amount of empirical evidence that higher rates of inflation are associated with increased variability and dispersion of relative prices. However, there is no theoretical model (nor empirical assessment) of the presumed link between this dispersion and the social cost of search.[2]

This paper provides such a model; more generally, it examines how anticipated inflation affects market efficiency, output and social welfare. This requires taking into account what all informal arguments, including the above quotation, omit: the impact of search on equilibrium prices and entry. Indeed, if increased search makes markets more competitive, then it may in fact not be a net waste from society's point of view.

1. Chapter 3, "The Costs of Inflation and Recession", page 79. The order of the first two sentences was inverted for clarity of the excerpt.

2. Fischer (1981) and Cuckierman (1982) provide surveys of the literature on inflation and price dispersion across goods. Domberger (1987) and Danziger (1987*b*) show similar effects within markets for homogenous goods. Fischer (1981) and Driffill, Mizon and Ulph (1989) review the main theories which could account for these correlations. The latter paper also reexamines some of the traditional evidence and questions whether it reliably establishes such links.

Previous models have captured some aspects of the interaction between inflation and market efficiency, but still miss others which are central to the issues at hand.[3] Benabou (1988) showed that in a search market with costs of price adjustment the increased price dispersion caused by inflation can intensify competition, reduce real prices and increase welfare (entry equalizes profits to zero). Diamond (1988), using different assumptions about search and price adjustment technologies, obtains similar effects for moderate inflation rates; but at higher rates, the exit of firms causes a worsening of the "thin-market" externality, and welfare declines as sellers become harder to find. These insights are useful, but restricted by two strong assumptions which both models share: buyers are identical, and have unit demands below a certain valuation for the good. The first restriction implies that in equilibrium buyers search only once, making the resource cost of search a non-issue. The second means that the only allocative role of prices is through entry. Moreover, because firms cannot overshoot consumers' reservation value, there is a sense in which the chosen specification of preferences makes it easier for inflation to decrease real prices than to increase them.

To address these issues, this paper generalizes Benabou (1988) in two directions:

(i) *Heterogeneity* among buyers accounts for search in equilibrium; the corresponding resource cost can then be linked to inflation.

(ii) Buyers with quite *general preferences* allow prices to play their full allocative role in the determination of output and welfare. Inflation can now potentially increase or reduce monopoly power and output, and the slope of this Phillips curve can be related to market structure.

In fact, one worthwhile purpose that this paper might serve is to dispel any (incorrect) perception that models of (S, s) pricing and search tend to imply that inflation is beneficial. With some of the specifications used, it can be quite harmful.

The generalizations described above make the equilibrium fairly complex. Firms follow staggered (S, s) rules, while buyers search sequentially. Those with a low search cost seek and find sellers have not revised their nominal price recently, while those with higher cost search less and buy more dearly. Correspondingly, a firm's sales increase while its nominal price remains fixed, and fall after each adjustment.

We prove the existence of equilibrium and identify the different components of welfare which are affected by inflation. In the case where costs of price adjustment are small, we show that inflation increases price dispersion and the resource cost of search, as traditionally asserted. For more general comparative statics, in particular those concerning output and welfare, we must resort to simulations. These confirm that the resource cost of search always rises with inflation, but also reveal that it remains quite small. The intuition is that the increased search by low search-cost buyers and the exit of firms both act as negative feed-backs which limit equilibrium price dispersion and total search. The output and welfare simulations indicate that what really matters is the way in which inflation alters monopoly power and the distribution of transaction prices. Two forces are at work there.

The first one is strategic complementarity between firms' prices, which reinforces the impact which inflation would have on a single monopolist's average price and output. This effect in turn can be positive or negative, depending on the form of consumers' preferences (Naish (1986), Konieczny (1989)). The second force is increased price

3. In focussing on the effects of inflation in a search market, we are leaving aside a parallel literature where imperfect competition arises instead from product differentiation (Danziger (1988), Konieczny (1987)). In these models as well, the net effect of inflation on welfare depends critically on consumer preferences and market structure.

dispersion, which always raises the return to search—particularly for low search-cost types. This tends to reduce monopoly power. Therefore, in markets where search is inexpensive (compared to consumer surplus from the good), welfare will not fall very much, and may even rise with inflation. When search is more costly, however, inflation can significantly decrease output and welfare.

The general conclusion of the paper is that formal analysis cuts both ways with respect to the conventional wisdom. It provides a rigorous basis for the idea that higher inflation results in more resources devoted to search, but also brings to light several more significant effects. These show that standard claims about the distortions which inflation causes in the price system can not be taken at face value; nor is the usual type of evidence on price dispersion sufficient to assess them. The theory points instead to the need for more focussed empirical studies, paying particular attention to inflation's impact on markups, the entry or exit of firms, and market structure in general.

Section 1 presents the model. Section 2 derives optimal strategies and entry. The existence of an equilibrium and some of inflation's effects are established in Section 3. Section 4 analyzes the different components of welfare, and their variation with inflation is examined through simulations in Section 5. Throughout the paper the reader might want to refer to Figure 1.

1. THE MODEL

1.1. *Overview*

It might be helpful to start with a general overview of the model's structure; specific assumptions will follow. We consider a monopolistically competitive search market under inflation. Its basic elements are illustrated in Figure 1, which is meant to serve as a road

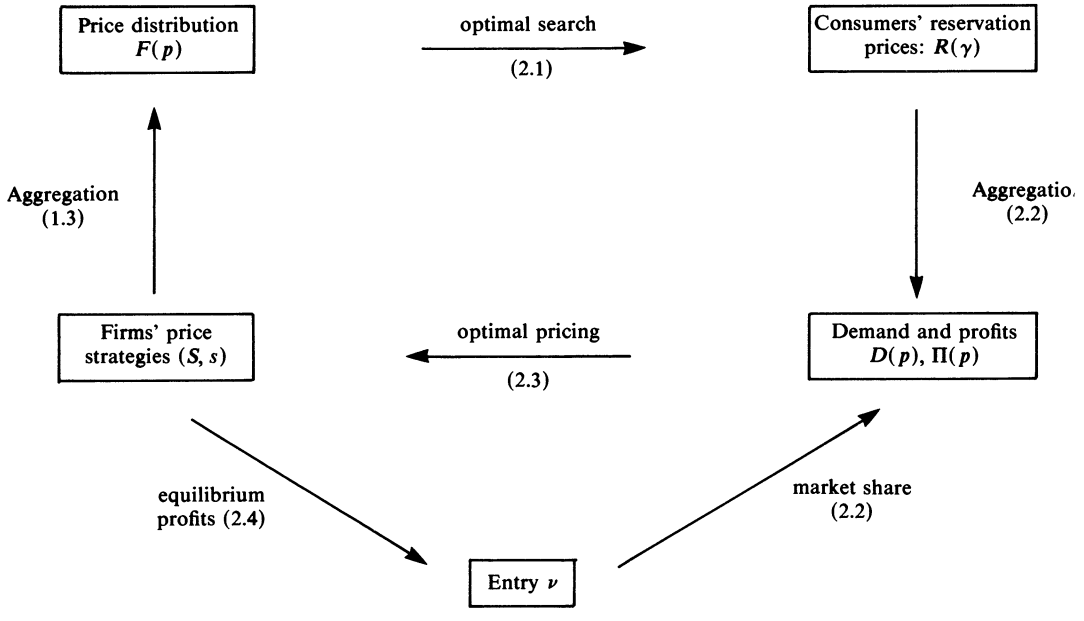

FIGURE 1

The structure of equilibrium. The numbers refer to the corresponding sections of the paper

map throughout the paper; indeed, each segment of the diagram corresponds to one of the sub-sections.

Firms set nominal prices, following staggered (S, s) price strategies. These generate a real price distribution $F(p)$, which determines an optimal reservation price $R(\gamma)$ for each buyer γ. These search rules aggregate to a demand curve $D(p)$ and profit function $\Pi(p)$, which in equilibrium must validate the original (S, s) price strategy as optimal for each firm. Finally, firms enter or exit the market until a marginal entrant's net present value of profits is zero.

We now describe in more detail the supply and demand sides of the market, as well as the inflationary process.

1.2. *Description of the market*

A homogeneous good is produced by a continuum of identical firms, using labour as the sole input. These firms are infinitely-lived, with discount rate ρ. During each unit of time, production requires a fixed cost $h \geqq 0$ and a marginal cost $c > 0$ per unit of output. Both are real costs, expressed in units of labour. Entry determines the measure $\nu \in (0, +\infty)$ of firms operating in equilibrium.

The demand side consists of a continuous flow of buyers who enter the market to make a one-time purchase of the good—for instance a durable. During a period of length dt, a measure $1 \cdot dt$ of them arrive. Their surplus or indirect utility from buying the good at a real price p is $V(p)$; so when they do buy at p, they purchase $z(p) = -V'(p)$ units. Since $z(p)$ is the demand they would address to a monopolist, it will be called monopoly demand.

Assumption 1. Consumer surplus $V(p)$ is decreasing, convex and twice-continuously differentiable inside its support $[0, M]$, $c < M \leqq +\infty$. Monopoly demand $z(p) \equiv -V'(p)$ has non-decreasing elasticity on $(0, M)$. If surplus is positive at any price ($M = +\infty$), then $\lim_{p \to \infty} [-pz'(p)/z(p)] > 1$.

Assumption 1 is quite weak, and satisfied by all usual demand functions (and associated surplus $V(p) = \int_p^\infty z(u)du$): reservation price, linear, iso-elastic, exponential, etc.[4] It implies the following well-known property:

Proposition 1. *The monopoly profit function $\pi(p) = (p - c)z(p)$ is strictly quasi-concave on its support, with a maximum at a finite $p^m \in (c, M]$.*

We consider, however, a market where buyers face many sellers whose prices they do not know, but among which they can search. Firms' demand and profit functions are then endogenous, and not equal to $z(p)$ and $\pi(p)$; Proposition 1 will nonetheless remain useful.

It is assumed that search is instantaneous and that buyers cannot postpone consumption; thus within the interval dt, all $1 \cdot dt$ of them arrive, search, buy, and leave the market. They are replaced by a similar generation of instantaneous consumers an instant later.[5] A first quotation is received for free but each subsequent search requires the expenditure

4. When $M < +\infty$, $z(p)$ may be discontinuous at M, or have an elasticity smaller than one on $[0, M]$.

5. Search is thus very fast with respect to real price erosion. This is the converse of the assumption in Cassella and Feinstein (1990), where inflation alters real prices faster than bargaining offers can be made. Indeed, theirs is a model of hyperinflation, while here we deal with moderate to high rates of inflation. We could also assume that search takes place in real time, but that consumers retain no memory of previous offers (whose current real value would otherwise be computable), so that they keep searching from the same distribution. Memory and intertemporal arbitrage would introduce a whole set of interesting but very complex issues into the model; see Benabou (1989) for some of them.

of γ units of labour.[6] Consumers' labour endowment is assumed to be large enough for income effects from search costs to be neglible. Thus each one maximizes the expectation of $V(p_k) - k \cdot \gamma$, where p_k is the (random) real price at which he ends up buying, after k costly searches. Consumers differ only by their search costs, and for simplicity we assume:

Assumption 2. Buyers' search costs are uniformly distributed on the interval $[\underline{\gamma}, \bar{\gamma}]$, with $0 \leqq \underline{\gamma} < \bar{\gamma}$.

1.3. *Inflation and* (S, s) *policies*

Money is used as the unit of account, so that firms must set their prices in nominal terms. We consider a regime of steady, perfectly anticipated inflation, in which all aggregate prices grow at a constant rate $g > 0$. This is true in particular of the nominal wage, with respect to which real prices are defined; a fixed nominal price thus means a real price falling at the rate g. Firms can change their nominal price at any time, but doing so entails a real fixed cost $\beta > 0$. This may stem from decision costs, changing price tags, issuing and sending new price lists and catalogues, etc. But β can also be viewed as a proxy for any adverse reactions of customers to a price increase, not captured by the model (say, reputation effects); these need not be small.[7]

A firm's optimal price policy in such an environment is generally an (S, s) rule: it adjusts its nominal price so as to achieve a real value of S, every time this real value has been eroded down to $s < S$.[8] Our focus will therefore be on equilibria where firms' optimal strategy is a common (S, s) rule; a more formal definition is given below.

The set of equilibria under consideration will be narrowed down further by the requirement that the cross-sectional distribution of firms' real prices be invariant over time. With a constant rate of inflation this corresponds to price adjustments which are uniformly staggered (Rotemberg (1983)), and the invariant distribution is log-uniform on $[s, S]$ (Caplin and Spulber (1987)):[9]

$$dF(p) = \frac{dp}{p \operatorname{Ln}(S/s)} \quad \text{for all } p \text{ in } [s, S]. \tag{1}$$

There are three reasons for this stationarity requirement. The first one is microeconomic consistency: a time-varying (non-degenerate) price distribution would result in non-stationary search rules, demand and profit functions.[10] This in turn would make stationary (S, s) price strategies suboptimal. The second one is macroeconomic consistency: only if the distribution of firms' real prices is stationary will their nominal prices

6. This standard assumption ensures that no consumer is kept out of the market because his expected surplus from search is less than the cost of the first sampling.

7. Although such costs generally increase with the size of price changes, the form of the optimal policy remains the same as long as the average cost $\beta(\Delta p)/\Delta p$ is decreasing; see Konieczny (1990).

8. See Sheshinski and Weiss (1977), Caplin and Shesinski (1987). The full specification of the strategy involves an additional trigger point $\hat{s} > S$, such that if $p > \hat{s}$, the firm adjusts down to S, but if $S < p < \hat{s}$ it lets inflation gradually do the job. Following the literature, we simply refer to the strategy as an (S, s) rule, although it implicitly includes an $\hat{s}$ which is relevant to describe out-of-equilibrium behaviour.

9. We thus focus on certain types of equilibria, namely staggered symmetric (S, s) equilibria, but do not impose any restriction on strategies. These equilibria are subgame-perfect, i.e. such that no firm ever wants to change the nature or timing of its price decisions, as long as no positive measure of firms or buyers have deviated. Other equilibria, e.g. synchronized (S, s), exist but do not satisfy macroeconomic consistency and feature no search.

10. For any distribution of search costs with low enough γ.

aggregate back into an index which grows smoothly, at the same rate g as the rest of the (macro) economy. The third reason is long-run stability. Almost any source of heterogeneity in firms' adjustment times, such as idiosyncratic cost or demand shocks, will cause the cross-sectional distribution of prices to converge to the steady-state distribution.[11]

1.4. *Entry*

In steady-state, the net present value of real profits for a potential entrant must be zero. As an entrant must pay β to set its first price (to S optimally), it is in the same position as a firm at s, about to start a new cycle. We can thus summarize the requirements illustrated on Figure 1 as follows:

Definition. A stationary, symmetric equilibrium is a triplet (S, s, ν) and a sequential search strategy for each buyer such that: (a) there are ν firms in the market, following identical, uniformly staggered (S, s) rules; (b) these price strategies and buyers' search strategies form a *subgame-perfect* equilibrium; (c) Firms earn zero net discounted profits over each price cycle.

2. SEARCH, DEMAND, AND PRICING IN EQUILIBRIUM

2.1. *Search*

We start by deriving each buyer's optimal search rule, given the distribution of prices (1) generated by firms' staggered (S, s) policies; see the upper part of Figure 1. For a buyer with search cost γ, the best strategy consists of accepting offers up to a real reservation price r at which he is indifferent between buying and searching again:[12]

$$V(r) = -\gamma + \int_s^r V(p)F(p)dp + \int_r^S V(r)F(p)dp, \tag{2}$$

assuming for now that this equation has a solution. Equivalently, the reservation price equates the cost and expected benefit of the marginal search:

$$\Gamma(r) \equiv \int_s^r [V(p) - V(r)]dF(p) = \int_s^r z(p)F(p)dp = \gamma. \tag{3}$$

The return $\Gamma(r)$ to searching rather than accepting a price r is increasing and continuously differentiable on $[s, M)$. Therefore, for a consumer with $\gamma < \Gamma(M)$, i.e. who would rather search than accept an offer leaving him with zero surplus, (3) has a unique solution $r = R(\gamma) \equiv \Gamma^{-1}(\gamma)$, with $s \leqq r < M$; this is his optimal reservation price. As seen from (2), the expected surplus of such a consumer in this market is $V(r) + \gamma$.

For a consumer with $\gamma > \Gamma(M)$, on the other hand, any offer below M is preferable to search, so let $R(\gamma) = M$. Of course in equilibrium no firm ever charges more than M, since this would lead to zero demand; thus $S \leqq M$ (this is formally proved in Section

11. Convergence occurs if firms are subject to cumulative idiosyncratic shocks or use different (S, s) bounds (Tsiddon (1987), Caballero and Engel (1989)), or if they randomize their price adjustments to deter storage by speculators (Benabou (1989)). While these results do not directly apply to an equilibrium model, where (S, s) rules are generally not optimal on the convergence path, their common intuition seems quite robust.

12. Since utility is linear in search expenditures and there is no limit to the number of searches a consumer can conduct (we shall return to this question later), optimal search is the same with and without recall.

2.3). The consumer will therefore accept the first offer encountered, and his expected surplus is just the market average:

$$\bar{V}(S, s) \equiv \int_s^S V(u)dF(u) = \int_s^M V(u)dF(u) = \int_s^M z(u)F(u)du = \Gamma(M) \tag{4}$$

where we used $S \leqq M$ and $V(M) = 0$. Summarizing both cases, a buyer with search cost γ rejects offers above his reservation price $r = R(\gamma)$:

$$R(\gamma) = \sup \left\{ r \in (s, M) \,\middle|\, \Gamma(r) = \int_s^r z(p)F(p)dp \leqq \gamma \right\} \tag{5}$$

which embodies both his preferences and search prospects. When faced with an offer $p \leqq R(\gamma)$, he accepts it and buys $z(p)$ units. The value of this optimal strategy is $V(R(\gamma)) + \Gamma(R(\gamma))$.[13] The highest and lowest reservation prices in the population, $R(\bar{\gamma})$ and $R(\underline{\gamma})$, will be denoted as $\bar{r}$ and $\underline{r}$.

2.2. *Demand and profits*

We now proceed to aggregate the search rules of individual buyers into the demand curve faced by firms, as indicated on Figure 1. Since no one buys above $\bar{r}$, we can focus attention on prices below $\bar{r}$. Moreover, intuition suggests that pricing above $\bar{r}$ is never optimal, so that $S \leqq \bar{r} \leqq M$; this is formally shown in the next section.

There are $1/(\bar{\gamma} - \underline{\gamma})$ buyers with given search cost γ. They search at random until they find one of the $\nu \cdot F(R(\gamma))$ firms charging a price $p \leqq R(\gamma)$, i.e. such that the marginal return to search $\Gamma(p)$ is less than their search cost γ. As a result, each of these firms will eventually retain $1/(\nu \cdot (\bar{\gamma} - \underline{\gamma}) \cdot F(R(\gamma)))$ such buyers, and each of them will buy $z(p)$ units. The demand for a firm charging a real price p is therefore:

$$D(p) = \frac{z(p)}{\nu} \cdot \left[\int_{\Gamma(p)}^{\bar{\gamma}} \frac{1}{F(R(\gamma))} \cdot \frac{d\gamma}{\bar{\gamma} - \underline{\gamma}} \right] \tag{6}$$

It is useful—and perhaps more intuitive—to express $D(p)$ in terms of the distribution of buyers' reservation prices. Consider first consumers with reservation price $r < M$; by (3), there are $\Gamma'(r)/(\bar{\gamma} - \underline{\gamma}) = z(r) \cdot F(r)/(\bar{\gamma} - \underline{\gamma})$ of them, and their probability of success in each round of search is $F(r)$. A given firm will therefore be visited by $z(r) \cdot F(r)/(\nu(\bar{\gamma} - \underline{\gamma}))$ of these buyers on their first search, $z(r) \cdot F(r)(1 - F(r))/(\nu(\bar{\gamma} - \underline{\gamma}))$ on their second search, $z(r) \cdot F(r)(1 - F(r))^{k-1}/(\nu(\bar{\gamma} - \underline{\gamma}))$ on their k-th search, etc.; hence a total of $z(r)/(\nu(\bar{\gamma} - \underline{\gamma}))$ consumers with reservation price r. Summing up over those who accept the firm's price, and adding the non-searchers ($r = M$, or $\gamma \geqq \Gamma(M)$), of which each firm gets a share $1/\nu$, we have:[14]

$$D(p) = \frac{z(p)}{\nu(\bar{\gamma} - \underline{\gamma})} \left[\int_p^M z(r)dr + \int_{\Gamma(M)}^{\bar{\gamma}} d\gamma \right] \quad \text{for } p \geqq \underline{r}, \tag{7}$$

or:

$$D(p) = \frac{1}{\bar{\gamma} - \underline{\gamma}} \frac{z(p)}{\nu} [V(p) + \bar{\gamma} - \Gamma(M)] \qquad \text{for } p \geqq \underline{r}. \tag{8}$$

13. If the first search was costly, the consumers with $\gamma > \Gamma(M)$ would stay out of the market. One might therefore be tempted to restrict attention to the case where $\Gamma(M) > \bar{\gamma}$, which also simplifies most equations and proofs. Such an equilibrium, however, does not always exist.

14. The argument leading to (7) implicitly assumes $\gamma \leqq \Gamma(M) \leqq \bar{\gamma}$. When $\Gamma(M) > \bar{\gamma}$, the bracketed term is just $\int_p^{\bar{r}} z(r)dr = V(p) - V(\bar{r})$. But $\Gamma(M) - \bar{\gamma} = (M) - \bar{\gamma} = \Gamma(M) - \Gamma(\bar{r}) = \int_{\bar{r}}^M z(r)F(r)dr = V(\bar{r})$ since $\bar{r} \geqq S$; hence (7) still holds. The general formula, including the case $\Gamma(M) < \gamma$, is given by equation (A1) in appendix.

For $p < \underline{r}$, $V(p)$ is replaced by $V(\underline{r})$: charging less than the lowest reservation price does not attract any more customers, since it does not affect the price distribution $F(p)$ on which search decisions are based. At prices $p \geqq \underline{r}$ search makes $D(p)$ more elastic than monopoly demand $z(p)$. Although this elasticity need not be increasing, the equilibrium demand function has the following fundamental property:[15]

Proposition 2. *If $\underline{r} \leqq p^m$, the equilibrium profit function $\Pi(p) = (p-c)D(p)$ is strictly quasi-concave on its support $[0, \bar{r}]$, with a maximum at some $p^* \leqq p^m$.*

Proof. See appendix. ‖

As in Caplin and Nalebuff (1988), the aggregation of heterogeneous individuals' demand functions results in a (strictly) quasi-concave profit function.[16] This property will play a crucial role in the existence of an equilibrium, ensuring both that a firm's optimal strategy is an (S', s') rule, and the continuity of this best response in (S, s) space.

2.3. *Price strategies*

We now close the price-search loop of Figure 1, by deriving a firm's best response to the (S, s) strategy of its competitors and buyers' search rules, as embodied in $D(p)$.

First, the strict quasi-concavity of the profit function $\Pi(p)$ is both necessary and sufficient to ensure that the optimal price strategy is a unique, stationary (S', s') rule (Zinde-Walsh (1986), Caplin and Sheshinski (1987))).[17] The firm's intertemporal real operating profits are therefore:

$$W(S', s') = \frac{\int_0^{T'} \Pi(S'e^{-gt})e^{-\rho t}dt - \beta}{1 - e^{-\rho T'}} \quad \text{with } T' = \frac{\text{Ln}\,(S'/s')}{g}. \tag{9}$$

T' is the duration of a real price cycle between S' and s', i.e. the length of time that the nominal price remains fixed. The firm chooses (S', s') or (S', T'), so as to maximize W. *If* there exists an interior optimum, it solves the first-order conditions $\partial W/\partial S' = 0$, $\partial W/\partial s' = 0$, or:

$$\Pi(s') = \rho W(S', s') \tag{10}$$

$$\Pi(S') - \Pi(s') = \rho\beta. \tag{11}$$

Condition (10) equates the benefit and opportunity cost of delaying adjustment when s' is reached: extra profits $\Pi(s')$. dt are earned, but the present value W is deferred by dt. Integrating (9) by parts, condition (11) can be rewritten as equating discounted marginal profits over the price cycle $\int_0^{T'} \Pi'(Se^{-gt})e^{-(\rho+g)t}dt$ to zero. In general the optimum

15. Note also that $D(p)$ may be discontinuous at $\bar{r}$, when $\bar{r} = M$, i.e. $\Gamma(M) < \bar{\gamma}$ and $z(M^-) > 0$ (e.g. unit demand below M); this implies $D(\bar{r}^-) > 0$.

16. In spite of the similarity, there is no obvious mapping of this search problem into a product differentiation model which would allow their powerful aggregation theorem to be used. A direct proof is thus required.

17. There are two additional wrinkles. First, profits must tend to zero as the price tends to $+\infty$; this holds under Assumption 1. Secondly, profits along this $(S's')$ path must remain positive, or else the firm will not always meet demand, as was implicitly assumed, and may even shut down. For a monopolist, a strategy with these properties (even one leading to a non-negative value of the firm) does not always exist—say if price adjustment costs are too large. Here, however, the entry or exit of firms will ensure that it always does.

need not be interior, that is, we may have $S' = M$.[18] It is shown in the appendix that the appropriate, more general form of the first-order conditions is:

$$\Pi(s') = \rho W(S', s') \tag{12}$$

$$\Pi(S') - \Pi(s') \geqq \rho\beta \quad \text{with equality unless (w.e.u.) } S' = M. \tag{13}$$

Conversely, *if* (12)-(13) have a solution, it is unique and is an interior optimum (Sheshinski and Weiss (1977)). In equilibrium, an individual price-setter's best response (S', s') must coincide with the (S, s) implemented by its competitors. It follows from the above discussion, and in particular from the uniqueness of the best response, that this fixed-point property is equivalent to the requirement that the original (S, s) pair solve (12)-(13). In particular, (13) requires $S' \leqq \bar{r}$, validating our previous claim that $S \leqq \bar{r} \leqq M$.

2.4. *Entry*

The last requirement for a monopolistically competitive equilibrium is that firms' operating profits (net of adjustment costs) just cover their fixed costs h/ρ:

$$\rho W(S, s) = h, \tag{14}$$

or

$$\Pi(s) = h \tag{15}$$

by (12) with $(S', s') = (S, s)$. It is worth noting from (15) that operating profits always cover fixed and variable costs, so firms are always willing to operate and satisfy demand. To illustrate the role played by entry and exit, suppose for instance that fixed costs h or adjustment costs β become large. Some firms then leave, increasing the remaining ones' market share $1/\nu$. As a result, the profits $\Pi(p) = (1/\nu) \cdot \pi(p)[V(p) + \bar{\gamma} - \Gamma(M)]/(\bar{\gamma} - \underline{\gamma})$ which they make at *any* price $p < \bar{r}$ rise, and so does the present value W given in (9). This continues until profits cover all costs.[19]

We can now express all equilibrium conditions in terms of $E \equiv (S, s, \nu, \underline{r}, \bar{r})$ only. In order to avoid rewriting the complicated expressions for the return to search, equilibrium profits and value from an (S', s') strategy, we shall simply make these functions' dependence on E explicit, by denoting them as $\Gamma_E(p)$, $\Pi_E(p)$, and $W_E(S', s')$ respectively. Γ_E is defined from (1) and (3); Π_E results from (8); finally, given Π_E, W_E is given by (9).

Proposition 3. *An equilibrium is a quintuple* $E \equiv (S, s, \nu, \underline{r}, \bar{r})$, *with* $s < S < +\infty$, $0 < \nu < +\infty$, and $\underline{r} \leqq p^m$, which solves:[20]

$$\Gamma_E(\underline{r}) \leqq \underline{\gamma}, \qquad \text{w.e.u.} \quad \underline{r} = M. \quad (\textit{Optimality of } \underline{r}) \tag{16}$$

$$\Gamma_E(\bar{r}) \leqq \bar{\gamma} \qquad \text{w.e.u.} \quad \bar{r} = M. \quad (\textit{Optimality of } \bar{r}) \tag{17}$$

$$\Pi_E(s) = h. \qquad (\textit{Optimality of } s) \tag{18}$$

$$\Pi_E(S) \geqq h + \rho\beta, \qquad \text{w.e.u.} \quad S = M. \quad (\textit{Optimality of } S) \tag{19}$$

$$W_E(S, s) = h/\rho. \qquad (\textit{Entry}) \tag{20}$$

18. In particular, when $\bar{r} = M$, $D(p)$ and $\Pi(p)$ may be discontinuous at $\bar{r}$ (see (8)), hence W not differentiable in S' at that point.

19. Equivalently, all fixed costs become smaller with respect to demand. There are some pricing costs (e.g. labelling goods) which increase with the volume of sales. But as long as the average cost per unit is decreasing, meaning that changing prices involves increasing returns, the results remains unaffected.

20. That $\underline{r} \leqq p^m$ is required in Proposition 2 for quasi-concavity; (18) and (19) imply $s \geqq c$ and $S \leqq \bar{r}$. The same system describes the equilibrium for a market without entry, if one fixes ν and lets h vary instead; such an equilibrium may fail to exist, i.e. (16)-(20) with a given ν may entail $h < 0$.

This system of five equations in five unknowns is the analytical counterpart to Figure 1. It summarizes the entry, pricing strategies and search decisions of all firms and buyers in the market. The fixed-point nature of an equilibrium is apparent from the fact that E appears as both parameter and argument.

3. EQUILIBRIUM

We first briefly examine a market with identical buyers. This simple case reveals most clearly an important mechanism by which inflation can promote competition via increased price dispersion. It also provides a robust intuition for the more complex case of heterogenous buyers, which is considered next. In that case the dispersion effect is counterbalanced and possibly dominated by other effects, but still plays an important role.

3.1. *Identical buyers*

Let buyers have the same search cost $\bar{\gamma} = \underline{\gamma} \equiv \gamma$ and unit demand, i.e. $V(p) = \max(M-p, 0)$. Since they have the same reservation price $\underline{r} = \bar{r} \equiv r$, the demand curve which firms face is just a step function:[21]

$$D(p) = 1/\nu \quad \text{for } p \leqq r, \qquad D(p) = 0 \quad \text{for } p > r. \tag{21}$$

Since demand is inelastic below r and zero above, firms optimally set $S = r$.[22] Equation (17) then becomes: $S - \int_s^S pdF(p) \leqq \gamma$ (w.e.u. $S = M$), so, with (18):

$$S = \min\left\{M, \gamma + \frac{S-s}{Ln(S/s)}\right\}; \qquad s = c + \frac{h}{\nu}. \tag{22}$$

The highest price is constrained by the minimum of consumers' willingness to pay and the average price in the market, plus the search cost; the lowest price reflects production costs and market share. Entry (20) determines ν. This is the model analyzed in detail in Benabou (1988); in particular:

Theorem 1. *If buyers have the same search cost and inelastic demand, there is a unique equilibrium. A higher inflation rate increases price dispersion S/s but reduces real prices S and s and the number of firms ν. Inflation improves consumer surplus and (at least when ρ is not too large) social welfare.*[23]

The basic intuition is very simple. In equilibrium all buyers accept the first price offered, but search matters as a credible threat constraining S. By increasing price dispersion and the return to search, a higher rate of inflation creates more competitive pressure on firms, forcing a lowering of the whole (S, s) price range and an increase in consumer surplus.[24] Due to staggering, the aggregate of net profits per unit of time is just ν times a single firm's net average profits over the cycle; when ρ is small, this is close to its net discounted profits over the cycle, which are zero due to entry. Thus total welfare increases as well.

21. Formally, replace $V(p)$ by $M-p$ in (8) and use l'Hôpital's rule to show that $(\bar{r}-\underline{r})/(\bar{\gamma}-\underline{\gamma})$ tends to $\Gamma'(r) = z(r)F(r) = 1$ as $[\bar{\gamma}, \underline{\gamma}]$ shrinks to $\{\gamma\}$ and $\bar{r}$ and $\underline{r}$ tend to a common limit r, with $\Gamma(r) = \gamma \leqq \bar{\Gamma}(M)$.

22. In this limiting case, (19) becomes $(S-c)/\nu \geqq h + \rho\beta$, w.e.u. $S = \bar{r}$, and one can show that (18) and (20) require that the inequality be strict.

23. Price dispersion is measured by $S/s - 1$. Benabou (1988) shows that the log-uniform distribution's coefficient of variation is a monotonic function of S/s. Danziger (1987*a*) proves a similar result for the distribution of purchase-weighted prices of a monopolist with isoelastic demand.

24. A decrease in γ also lowers S, s and ν but reduces price dispersion S/s.

To show that inflation can also *lower* welfare by increasing monopoly power and generating a resource cost of search, we now turn back to the general model. The reader who wishes to skip the derivation of the equilibrium can go directly to Theorem 2 below.

3.2. *Heterogeneous buyers*

Let us revert to the case of different buyers with general preferences satisfying Assumption 1, and simplify the problem by assuming that $\underline{\gamma}=0$. Condition (16) then gives $\underline{r}=s$, which must be no greater than p^m. Condition (17) is unchanged and determines the maximum reservation price $\bar{r}$ as a function of (S, s). Condition (18) gives the equilibrium number of firms:

$$\nu=\frac{\pi(s)}{h\bar{\gamma}}[V(s)+\bar{\gamma}-\bar{V}(S, s)] \tag{23}$$

where $\bar{V}(S, s)=\Gamma(M)$ by (4). Substituting ν into (19) and (20) then leads to a system in the two unknowns (S, s) only:[25]

$$\pi(S)[V(S)+\bar{\gamma}-\bar{V}(S, s)] \gtreqless \left[1+\frac{\rho\beta}{h}\right]\pi(s)[V(s)+\bar{\gamma}-\bar{V}(S, s)], \quad \text{w.e.u.} \quad S=M; \tag{24}$$

$$\int_s^S \frac{\pi(u)[V(u)+\bar{\gamma}-\bar{V}(S, s)]}{\pi(s)[V(s)+\bar{\gamma}-\bar{V}(S, s)]}\, u^{\delta-1}du=\left[\frac{g\beta\cdot S^\delta}{h}+\frac{S^\delta-s^\delta}{\delta}\right] \tag{25}$$

where $\delta\equiv\rho/g$. This system completely determines the equilibrium, provided $s\leqq p^m$ and the right-hand side of (24) is positive. It shows that equilibrium prices depend on the market frictions only through the ratios β/h of adjustment costs to fixed costs, and $\bar{\gamma}/V(.)$ of search costs relative to consumer surplus. We shall come back to these intuitive properties when interpreting the simulations of Section 5.

Using in particular the strict quasi-concavity of Π established in Proposition 2 and the properties of $z(p)$ from Assumption 1, we prove:

Theorem 2. *When $\underline{\gamma}=0$, there exists an equilibrium; buyers search actively and each firm's sales are cyclical.*

Proof. See appendix. ‖

This result is of some independent interest because it provides a parallel, for search and dynamic price-setting, to the general existence results of Caplin and Nalebuff (1988) for static models of imperfect competition. In both cases the equilibrium rests on the property that the demands of individuals with appropriately distributed characteristics aggregate to a quasi-concave profit function, so that firms' best-reply correspondence is continuous.

In general one cannot establish the uniqueness of the (symmetric) equilibrium analytically, but numerical simulations strongly suggest that such is the case.[26] In any event, all equilibria share the same basic features: buyers with low search actively seek

25. These conditions assume $\delta>0$ and $h>0$. Similar ones hold when there is no discounting or no fixed costs. When $\rho\to 0$, $(S^\delta-s^\delta)/\delta$ tends to $\mathrm{Ln}(S/s)=T/g$, and firms maximize profits per unit of time, i.e. the limit of ρW, as given by (9). When $h=0$, (18) implies that $s=c$.

26. There may also be non-symmetric equilibria, where different groups of firms use different (S, s) bands and equilibrium profits are not strictly quasi-concave; these lie outside the scope of this paper.

firms which have not revised their nominal price recently—and are thus at a low point of their (S, s) cycle. Buyers whose search cost is higher search less and do not fare as well. As a result, a firm's sales increase while its nominal price remains fixed, and fall after each adjustment.

3.3. *The resource cost of search*

We can now evaluate total search expenditures by consumers C_s, and see whether they rise with inflation. To compute C_s, note that each buyer with cost γ searches on average $1/F(R(\gamma))$ times; since the first one is free, we have[27]

$$C_s = \int_{\underline{\gamma}}^{\bar{\gamma}} \gamma \cdot \left[\frac{1}{F(R(\gamma))} - 1\right] \frac{d\gamma}{\bar{\gamma} - \underline{\gamma}} = \int_{\underline{r}}^{\bar{r}} \Gamma(r) \left[\frac{1}{F(r)} - 1\right] \Gamma'(r) \frac{dr}{\bar{\gamma} - \underline{\gamma}},$$

or[28]

$$C_s = \int_{\underline{r}}^{\bar{r}} z(r)\Gamma(r) \frac{dr}{\bar{\gamma} - \underline{\gamma}} - \frac{\bar{\gamma} + \underline{\gamma}}{2} \tag{26}$$

We now show that price dispersion and the cost C_s increase with the inflation rate, assuming the costs of price adjustment are small. Consider first the case where prices are perfectly flexible, i.e. $\beta = 0$. The equilibrium then reduces to a single real price $S = s = p_*$. By Proposition 2, p_* is the unique maximizer of $\Pi_*(p) = \pi(p)[V(p) + \bar{\gamma} - V(p_*)]$, since $\bar{V}(p_*, p_*) = V(p_*)$. Note that the threat of search generally forces $p_* < p^m$. Now, for small $\rho\beta/h$ and $g\beta/h$, third-order Taylor expansions allow us to show:

Proposition 4. *Assume that price adjustment costs are small* ($\beta \ll \min\{h/\rho, h/g\}$), *and* $\underline{\gamma} = 0$. *A higher rate of inflation then results in more frequent price adjustments, increased real price dispersion and more resources spent on search. Specifically*:

$$\frac{S - p_*}{p_*} \approx \frac{p_* - s}{p_*} \approx \frac{gT}{2} \approx \left[\frac{g\beta}{ah}\right]^{1/3} \tag{27}$$

$$C_s \approx \left[\frac{g\beta}{bh}\right]^{2/3} \tag{28}$$

where p_* *is the real price when* $\beta = 0$, $a = -2p_*^2 \cdot \Pi_*''(p_*)/(3\Pi^*(p_*))$ *reflects the concavity of equilibrium profits near* p_*, *and* $b = a \cdot (6\bar{\gamma})^{3/2}/(p_* \cdot z(p_*))^3$.

Proof. See appendix. ‖

This result is interesting because one often thinks of costs of price adjustment as not being very large. While the approximations require in principle that $(g\beta/h)^{1/3}$ and $(\rho\beta/h)^{1/3}$ be small, hence $g\beta/h$ and $\rho\beta/h$ very small, simulations show that they remain reliable up to $\beta/h \leqq 0{\cdot}1$ and $g \leqq 50\%$.[29] On the other hand, this still leaves out interesting

27. Because of his budget constraint, each buyer's labour endowment L should cover his search costs. This cannot be ensured with probability one, but the probability that a buyer runs out of resources before finding $p \leqq R(\gamma)$ is very small if $L \gg \gamma[1/F(R(\gamma)) - 1]$. For all $r \geqq s$, (3) implies $\Gamma(r) \leqq (V(s) - V(r))F(r)$, so that $\gamma/F(R(\gamma)) \leqq V(s)$ and it suffices to assume that $L \gg V(c)$.

28. For simplicity, it is assumed that $\bar{r} < M$. Otherwise, the last term in (26) must be replaced by $(\Gamma(M)^2 - \gamma^2)/(2(\bar{\gamma} + \gamma))$.

29. Dixit (1991), examining models of (S, s) behaviour under uncertainty, also finds Taylor approximations to be fairly reliable.

cases, such as free entry ($h=0$); more generally, realism only requires β to be small compared to gross revenue $p \cdot D(p)$, not to operating profits $(p-c)D(p)$, or $h=(s-c)D(s)$.

In any case, the resources spent on search are only one of the components of welfare; we now turn to the others, proceeding in two stages. In Section 4 output, surplus, profits and welfare are computed as functions of the equilibrium $E=(S, s, \bar{r}, \underline{r}, \nu)$; this brings to light the different channels through which inflation operates. As the model is too complicated to do comparative statics and assess the importance of the various effects analytically, simulations are then performed in Section 5.

4. INFLATION AND WELFARE

Welfare is defined as the sum of aggregate consumer and producer surplus, and not in any Paretian sense. Indeed it will be clear that inflation causes substantial redistributions between firms and consumers, as well as among consumers with different search costs. Turning to buyers first, their welfare per unit of time is simply the value of their search strategies:

$$B_c = \int_{\underline{\gamma}}^{\bar{\gamma}} [V(R(\gamma))+\Gamma(R(\gamma))] \frac{d\gamma}{\bar{\gamma}-\underline{\gamma}} \tag{29}$$

which depends only on the equilibrium (S, s) bounds. In particular, if buyers have unit demands, as in Benabou (1988) or Diamond (1988), all that matters is their common reservation price, i.e. the highest price in the market. A more fruitful decomposition of B_c is the difference between the gross surplus from all transactions and the total resource cost of search C_s:

$$B_c = \int_s^S V(p)N(p)dF(p) - C_s \tag{30}$$

where $N(p)=D(p)/z(p)=\min(V(p), V(\underline{r}))+\bar{\gamma}-\bar{V}(S, s))$ is the total number of transactions at a price p, and $N.dF$ sums to one on $[s, S]$.[30] Aggregate output is given by the same expression as gross surplus B_c+C_s, but with $V(p)$ replaced by $z(p)$. Clearly, inflation will affect surplus and output both through F, the *distribution of prices* in the market, and through N, the *distribution of transactions* across prices. This latter effect is accompanied by a change in the total amount of search and its cost C_s, given by (26).

In the absence of discounting, B_c would represent total social welfare. Indeed, as price revisions are uniformly staggered, aggregate profits per unit of time coincide with a given firm's average profits over the (S, s) cycle, times the number of firms ν; when $\rho=0$, these average profits, net of fixed and menu costs, are equalized by entry to zero (let $\rho \to 0$ in (9) and (14)).

When $\rho>0$, undiscounted profits over the cycle are larger than discounted profits; since it is the latter (net of fixed costs) which are zero, total profits per unit of time are positive, and can be decomposed into:

$$B_f = \int_s^S (p-c)z(p)N(p)dF(p) - \nu\left[h+\frac{\beta}{T}\right] \tag{31}$$

30. The equality of (29) and (30) follows from (26) and the definition of Γ.

The first term is operating profits from the $N(p)$ transactions at each firm; the second one is total fixed costs; the last one is total adjustment costs, since a proportion $1/T = g/\text{Ln}\,(S/s)$ of firms adjust per unit of time.[31] Summing up, social welfare equals *aggregate gains from trade*, minus the resources spent on the three types of *market frictions*: search, price adjustment, and fixed operating costs:

$$B = \int_s^S [(p-c)z(p) + V(p)]N(p)dF(p) - C_s - \nu\left[h + \frac{\beta}{T}\right]. \tag{32}$$

The first term reflects the allocative role played by prices through production, search and consumption decisions. With unit consumer demand it becomes a constant. In general, however, the equilibrium (S, s) price cycles can either worsen or alleviate the inefficiency of monopolistic pricing, and this term will thus be affected by inflation.

The second term reflects the resources spent on search for better prices, and will generally rise with inflation. It vanishes in models with identical buyers, since all accept the first offer received.[32] The third effect of inflation will be to tend to reduce the number of firms in the market (by forcing them to charge less profitable real prices and adjust more often), and with it the inefficient duplication of fixed costs which characterizes monopolistic competition when $h > 0$. Finally, this induced exit will partially offset the increase in adjustment costs per operating firm (last term).

5. SIMULATIONS

We now turn to simulations of the model. Their purpose is not to match actual data, but simply to help explore the various channels through which inflation operates in equilibrium, and provide a sense of the magnitudes involved. Two specifications of preferences are used, corresponding to isoelastic ($z(p) = p^{-\alpha}$, $M = +\infty$) and linear ($z(p) = M - p$) monopoly demand. The inflation rate g ranges from zero to fifty per cent a year. A large number of parameter values were explored, but only a dozen representative outcomes are reported. The reference set of parameters is $c = 1$, $\underline{\gamma} = 0$, $\bar{\gamma} = 0{\cdot}10$, $\beta = h = 10$, $\rho = 0{\cdot}05$, $\alpha = 4{\cdot}1$ or $M = 8$; alternative values are explored in Figures 2(a) to 8(c). The parameters were chosen so as to yield plausible values of the duration of prices, price dispersion, and adjustment costs as a fraction of revenues over a cycle.[33]

5.1. *Price dispersion, search, and exit*

Three very *robust results* come out of the simulations, holding across all preference specifications and parameter values. They show in particular that the conclusions of Proposition 4 hold quite generally, and not just for very small price adjustment costs.

31. Alternatively, each firm's intertemporal profits, starting from s, are zero; but they are positive until it reaches s from its initial position, i.e. until its first adjustment. Thus (equality with (31) rests on (1) and (20)):

$$B_f = \nu \int_s^S \int_0^{\theta(p)} [\Pi(pe^{-gt}) - h]e^{-\rho t}dt\,dF(p) \quad \text{where } \theta(p) = \text{Ln}\,(p/s)/g.$$

32. In Diamond (1988) there is a cost of involuntary search, which also depends on inflation. Impatient consumers must wait for buying opportunities, whose arrival rate depends on the stock of goods produced but yet unsold; the latter is determined by a zero-profit condition. It is as if the first term of (32) were multiplied by an increasing function of ν. We could incorporate a similar thin-market effect here by having the distribution of search costs shift up as ν falls.

33. The latter vary between 7% at $g = 0\%$ and 0·05% at $g = 50\%$. Since $\beta/h = 1$, the same holds for fixed costs. This small value of h is meant to reflect monopolistic competition; higher values are explored in the simulations.

Result 1. *As the rate of inflation increases, so does real price dispersion, although prices are changed more frequently.*

For the isoelastic specification (Figure 2(*a*)), both price dispersion and its rate of increase with inflation are quite significant: for g equal to 1%, 10% and 50%, $S/s-1$ is respectively 9·3%, 19·5% and 31·7%. This accords well with the empirical evidence that higher rates of inflation are associated with greater price dispersion, such as Fischer (1981), Domberger (1987), or Danziger (1987*b*). The linear specification, on the contrary, leads to a very weak relationship (Figure 2(*b*)): price dispersion only rises from 1·9% at $g=1\%$ to 2·8% at $g=50\%$, while the decrease in T is very fast: for $g=1\%$, 10% and 50%, prices are revised every 20·6, 5·7 and 0·7 months, versus every 97·0, 21·2 and 6·6 months under the isoelastic specification. This is consistent with Kashyap's (1986) findings of very small price adjustments (2-3% for certain goods). Cecchetti (1986) also shows that for magazine prices, higher inflation is accompanied by more frequent but not significantly larger adjustments. In his case, however, $S/s-1$ is always large (25%); this type of behaviour seems more difficult to account for with the type of (S, s) model considered here.

Figures 2(*a*) and 2(*b*) also show firms' desired price p^* and the average transaction price $\tilde{p}$, which indicates how effective searchers are in finding the "bargains" which inflation creates in the market.[34,35]

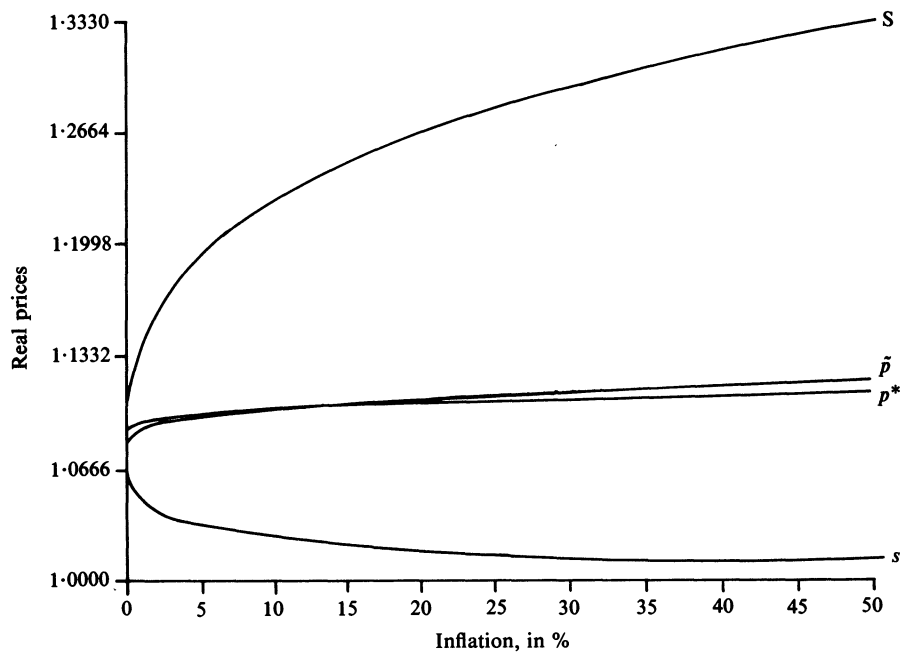

FIGURE 2(*a*)

Inflation and real prices. $c=1, \beta=h=10, \bar{\gamma}=0{\cdot}10, \rho=0{\cdot}05, z(p)=p^{-\alpha}, \alpha=4{\cdot}1$

34. In $\tilde{p}$, prices are weighted by the number of buyers who end up paying them, but not by the quantity bought. A purchase-weighted average would essentially vary inversely with output, which is examined later.

35. The equilibrium without inflation is also noteworthy. When $g=\rho=0$, firms adjust once and for all to the unique p_* which maximizes profits per unit of time. Since $S=s=p_*$ there is no search, but the threat of search by buyers with low γ forces $p_*<p^m$. When $\rho>0$, there is a discontinuity at $g=0^+$: because the discrete cost β must be compensated by a discrete increase in discounted profits, S and s remain bounded away from one another, generating price dispersion and search; indeed, (13) requires that $S>s$.

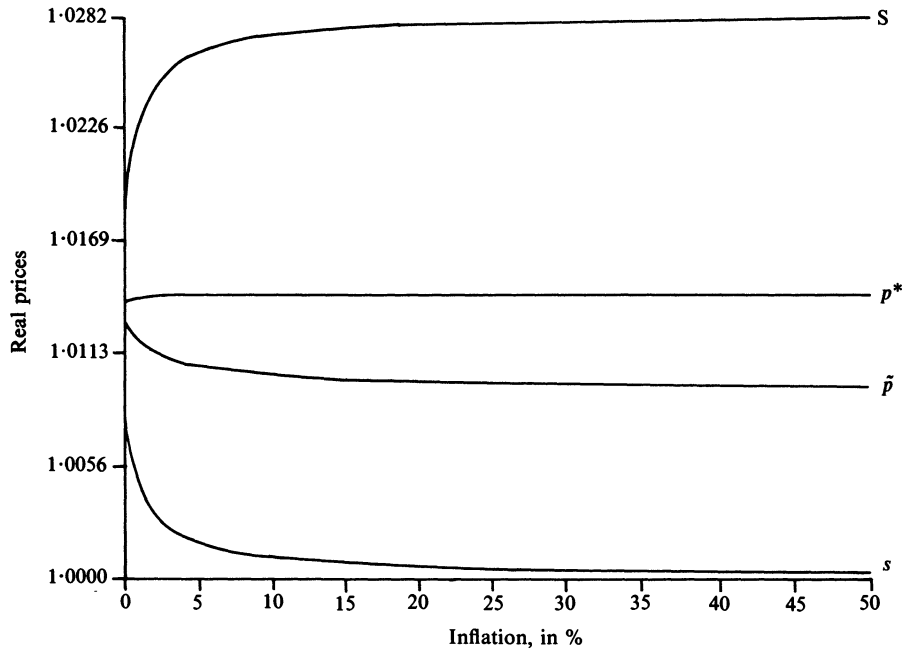

FIGURE 2(*b*)

Inflation and real prices. $c = 1$, $\beta = h = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p) = M - p$, $M = 8$.

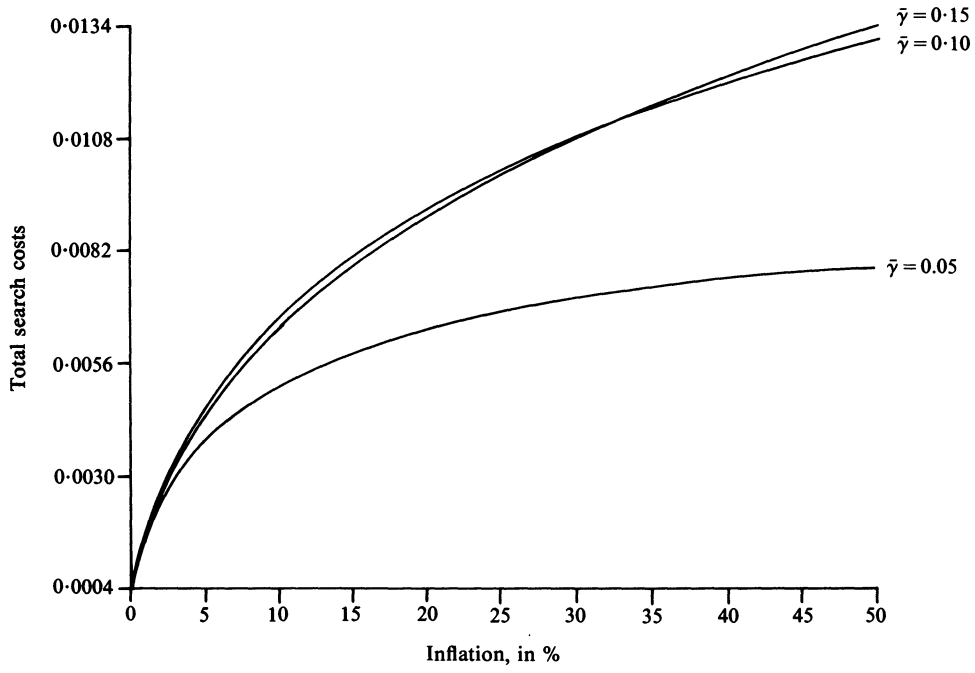

FIGURE 3(*a*)

Inflation and the resource cost of search. $c = 1$, $\beta = h = 10$, $\rho = 0{\cdot}05$, $z(p) = p^{-\alpha}$, $\alpha = 4{\cdot}1$

The next result, illustrated in Figures 3(a) and 3(b), confirms the idea of a cost of inflation due to increased search, but with an important caveat.

Result 2. *As the rate of inflation increases, so do the total number of searches and the total resources spent on search. This cost, however, remains small.*

The basic mechanism is clear: although inflation may cause prices to be higher or lower on average (see p^* and $\tilde{p}$), it always increases dispersion, and thereby results in increased search and more resources spent on search.[36]

There is, however, an opposing force at work: while more price dispersion generates more search, more search intensifies price competition and allows less price dispersion.[37] This is why C_s increases rapidly with inflation rates of up to about 20%, then tapers off. As a result, the total cost of search remains rather small; on average consumers search only a couple of times. This result is very robust to variations in $\bar{\gamma}$ and other parameters, and perhaps not as surprising as one would initially think. If search costs are high, increased price dispersion at higher inflation rates is sustainable, but only because few consumers can take advantage of it; if search costs are low, firms will not allow large price dispersion even at high inflation, so as to limit the amount of search by buyers.

The next result (and Figures 4(a)-4(b)) shows that the exit of firms constitutes a second feedback which limits the impact of inflation on real prices. In fact, the effects

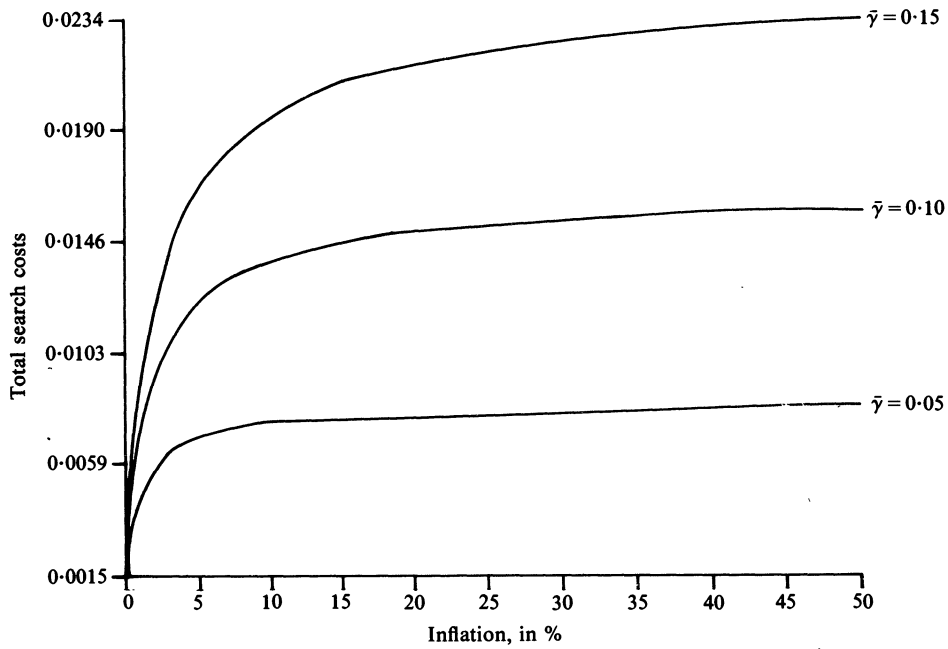

FIGURE 3(b)

Inflation and the resource cost of search. $c = 1, \beta = h = 10, \rho = 0{\cdot}05, z(p) = M - p, M = 8$

36. Buyers with low search cost can be shown to always search more; the behaviour of those with high search costs is ambiguous, but all simulations show that the total number of searches increases with inflation, in a manner similar to the total cost of search.

37. This interaction was noted by Paroush (1986) in his discussion of the effects of inflation.

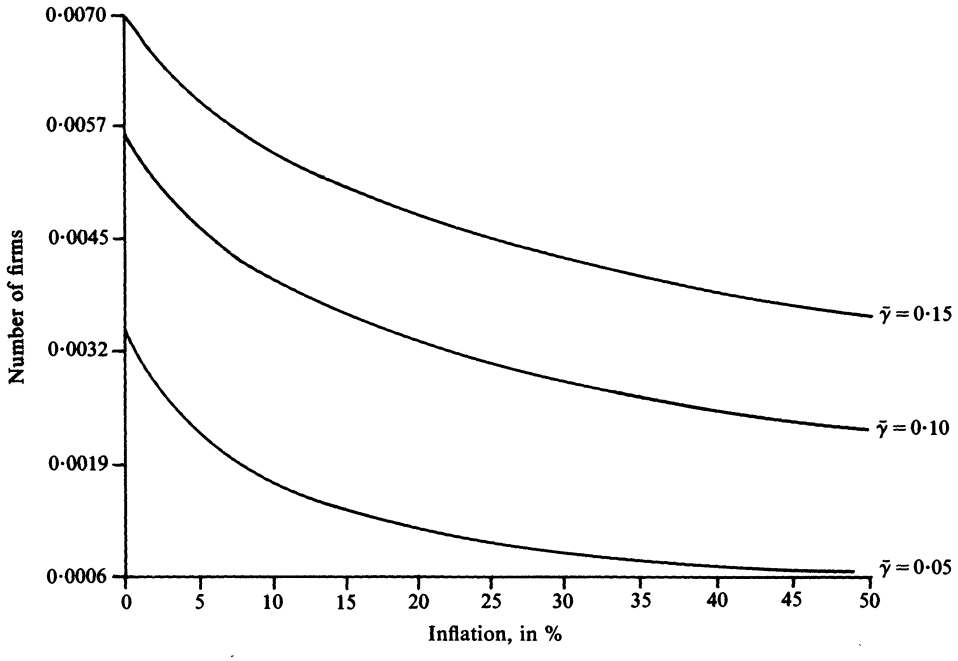

FIGURE 4(*a*)

Inflation and the number of firms. $c = 1$, $\beta = h = 10$, $\rho = 0{\cdot}05$, $z(p) = p^{-\alpha}$, $\alpha = 4{\cdot}1$

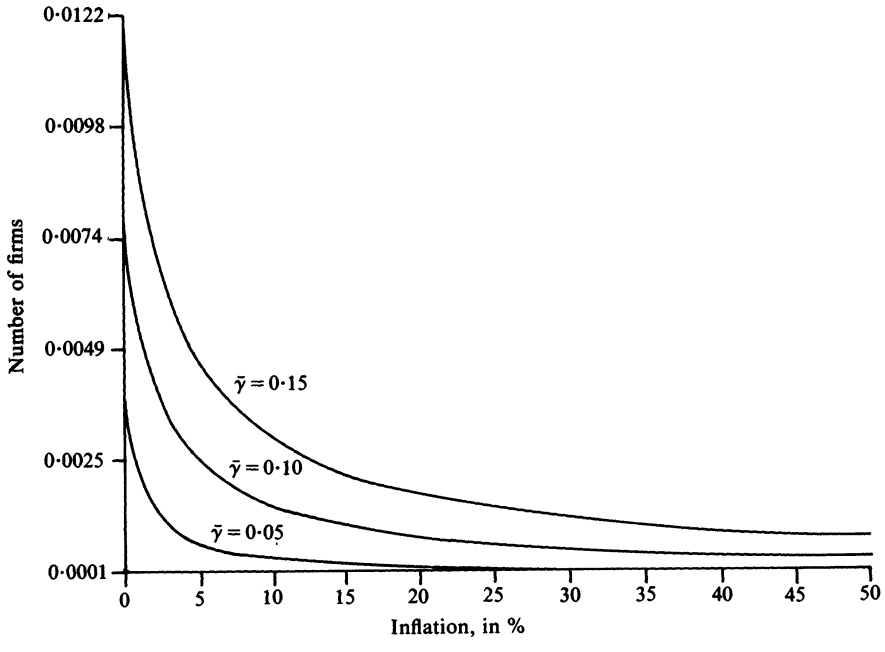

FIGURE 4(*b*)

Inflation and the number of firms. $c = 1$, $\beta = h = 10$, $\rho = 0{\cdot}05$, $z(p) = M - p$, $M = 8$

of search and exit as dampening forces are visible on all equilibrium variables (most curves become fairly flat above $g = 20\%$). Because informal arguments fail to take them into account, they may well overestimate both the costs and benefits of inflation.

Result 3. *As the rate of inflation increases, the number of firms in the market decreases.*

Higher inflation forces each firm to widen its price range around p^* (Result 1); this always lowers profits, as do the more frequent menu costs. Equilibrium profits are restored by the exit of some firms; the remaining ones' increased market share now makes it profitable for them to revise their prices more often. This constitutes the second limiting force on price dispersion and search.

The model's implication that inflation results in markets which are more concentrated (but not necessarily less competitive, as will be seen below) could be tested by checking whether prolonged inflationary periods are associated with greater numbers of bankruptcies and mergers. The elimination of firms from a monopolistically competitive market promotes efficiency by reducing rent dissipation, i.e. the duplication of fixed and pricing costs. On the other hand, bankruptcies entail economic and political costs (reorganization, unemployment) which might realistically offset this efficiency gain, at least from an elected policy-maker's point of view.

We now turn to the effects of inflation on output (or on the average transaction price $\tilde{p}$) and on the different components of welfare. Unlike the preceding ones, they turn out to be very sensitive to the specification of consumers' *preferences* and the values of the *market frictions* $\bar{\gamma}$, β and h.

5.2. *The inflation-output relationship*

Naish (1986) and Konieczny (1989) have examined whether inflation increases or lowers the average price and output of a monopolistic firm over its (S, s) cycle. The answer unfortunately depends on two somewhat unintuitive factors: the skewness of the (log) profit function, which determines whether S increases faster or slower than s decreases as inflation rises, and the curvature of the demand function, which determines the output consequences of these changes (through Jensen's inequality). With constant marginal cost, inflation tends to decrease output if demand is iso-elastic, and to increase it if demand is linear (Naish (1986)). This sensitivity in itself justifies modelling preferences with enough generality.

Under monopolistic competition, demand and profit functions are endogenous, and therefore affected by inflation; this makes skewness and curvature more elusive concepts. It remains true, however, that the functional form of $z(p)$ strongly influences demand $D(p)$ and profits $\Pi(p)$; see (8). There is unfortunately no more economic intuition to be gained here than in the monopoly case. But equally important are other firms' pricing and buyers' purchasing decisions, embodied in the number of buyers at a price p, namely $D(p)/z(p) = [V(p) + \bar{\gamma} - \bar{V}(S, s)]/(\nu\bar{\gamma})$. These decisions reflect in turn the whole market structure, i.e. the costs of search, price adjustment, and entry. It is on these equilibrium aspects of the inflation-output relationship that the model will deliver some new results and intuitions.

Figures 5(a)-5(b) and 6(a)-6(b) plot *percentage* deviations of output from its zero inflation level. They confirm that the monopoly results need no longer hold, but only provide a reasonable "first guess". With the iso-elastic specification, the effect of inflation can be *strongly negative*: a rise in g from zero to 5% a year reduces output by 4.0%; from

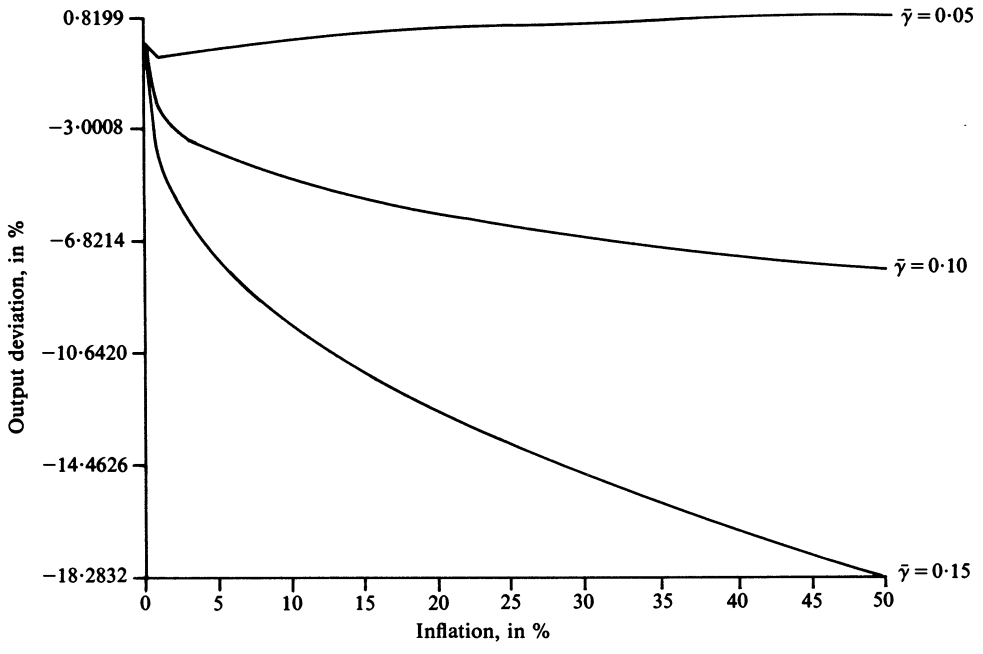

FIGURE 5(*a*)

Inflation, output, and search costs. $c = 1$, $\beta = h = 10$, $\rho = 0{\cdot}05$, $z(p) = p^{-\alpha}$, $\alpha = 4{\cdot}1$

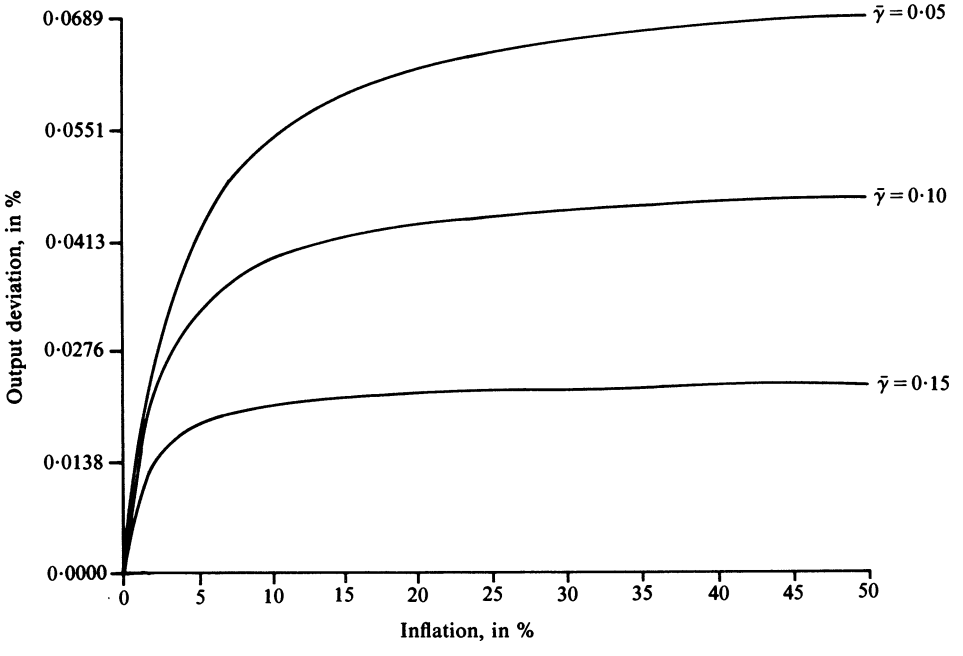

FIGURE 5(*b*)

Inflation, output, and search costs. $c = 1$, $\beta = h = 10$, $\rho = 0{\cdot}05$, $z(p) = M - p$, $M = 8$

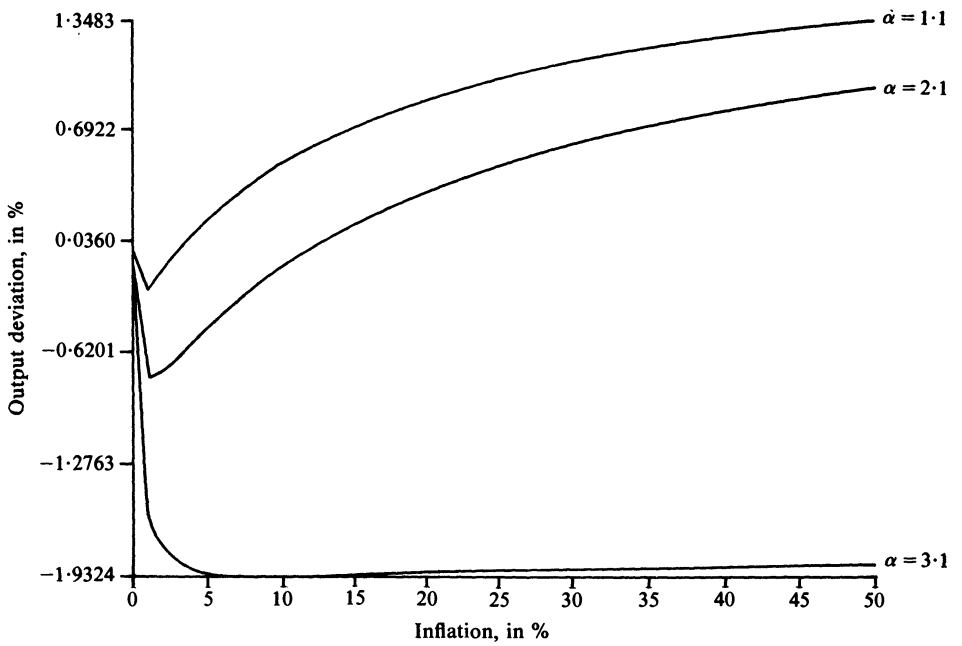

FIGURE 6(*a*)

Inflation, output, and preferences. $c = 1$, $\beta = h = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p) = p^{-\alpha}$

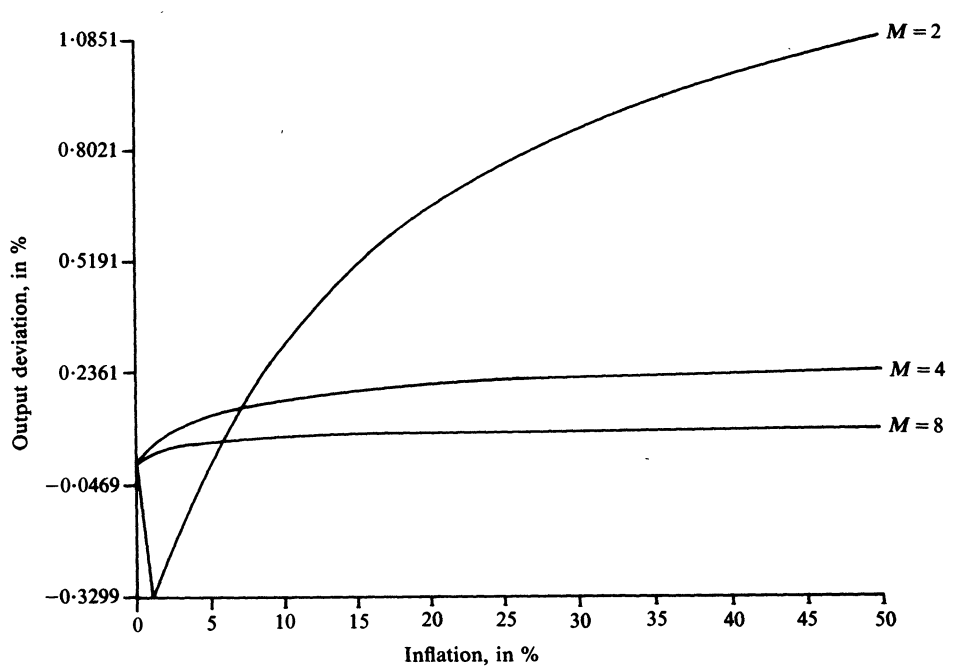

FIGURE 6(*b*)

Inflation, output, and preferences. $c = 1$, $\beta = h = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p) = M - p$

5% to 10% there is a further decrease of 3·5% (Figure 5(a) for $\bar{\gamma} = 0{\cdot}15$). With a lower α, the slope of the Phillips curve becomes slightly positive; for $\alpha = 1{\cdot}1$, a 10% inflation rate raises output by 0.5% (Figure 6(a)). With the linear specification, the impact of inflation is less than 2% either way (Figures 5(b) and 6(b)); as mentioned before, it mostly translates into a rapid decrease in T and ν.

To understand the intuition for what determines the average price and output in equilibrium, assume first that in response to an increase in inflation, firms re-set their (S, s) bands as monopolists, not taking into account the fact that their profit and demand functions will be different in the new equilibrium. The (S, s) band widens, and the average price moves up or down, depending on the form of the old $\Pi(p)$. But both these effects have equilibrium consequences. First, if firms now have higher prices on average, then any one of them can shift its price band up a little without losing too many customers; this in turn encourages other firms to move still a little higher, and so on. The same holds true for downward movements. Thus *strategic complementarity* reinforces a single firm's incentive to shift its (S, s) band in either direction. Secondly, the increased dispersion per se always raises the return to search, as in Theorem 1, and this works to lower prices. The relative strength of these two forces depends in particular on how easy it is to search, and on how much surplus is derived from finding a lower price. This intuition is confirmed by the simulations shown on Figures 5(a)–5(b).

Result 4. *A higher rate of inflation can increase or decrease output. The slope of this Phillips curve depends on preferences and market structure. If search costs are low (respectively, high) relative to consumer surplus, inflation tends to raise (respectively, lower) output.*

Indeed, the more desired the good (low α or high M) and the easier it is to search (low $\bar{\gamma}$), the more demand and profits are determined by search as opposed to tastes, as more buyers try to take advantage of increased price dispersion to find low prices. In the converse situation, severe stagflation is possible. The model thus delivers positive results in spite of the sensitivity of the Phillips curve to the functional form of consumer surplus (itself a reminder of the risks of drawing conclusions from convenient but restrictive specifications). In particular, the implication that inflation has a differential impact on markups and output across markets with different degrees of friction or competitiveness should be empirically *testable.*

As shown by Figure 7, the entry cost h also significantly affects the slope of the output-inflation locus. Higher fixed costs imply fewer operating firms; their larger market shares then justify more frequent price revisions, and this reduces real prices' sensitivity to inflation.

5.3. *Surplus, profits, and welfare*

Figures 8(a)–8(b) illustrate the typical results emerging from welfare simulations. Social welfare generally varies like its main component, gross consumer surplus, which in turn follows output. It is possible, however, that increased search costs reverse this relationship (Figure 8(c), for g between 1% and 4%). The original concern with this cost of inflation thus had some validity, although the numbers here remain quite small. For firms, gross profits vary inversely with output (as does the average transaction price $\tilde{p}$). The resources spent on fixed costs decrease as firms are forced out of the market, but the total cost of price adjustments increases. In fact, net profits always decrease (when $\rho > 0$), subtracting from total welfare. As before, the effects are much larger with iso-elastic than with linear

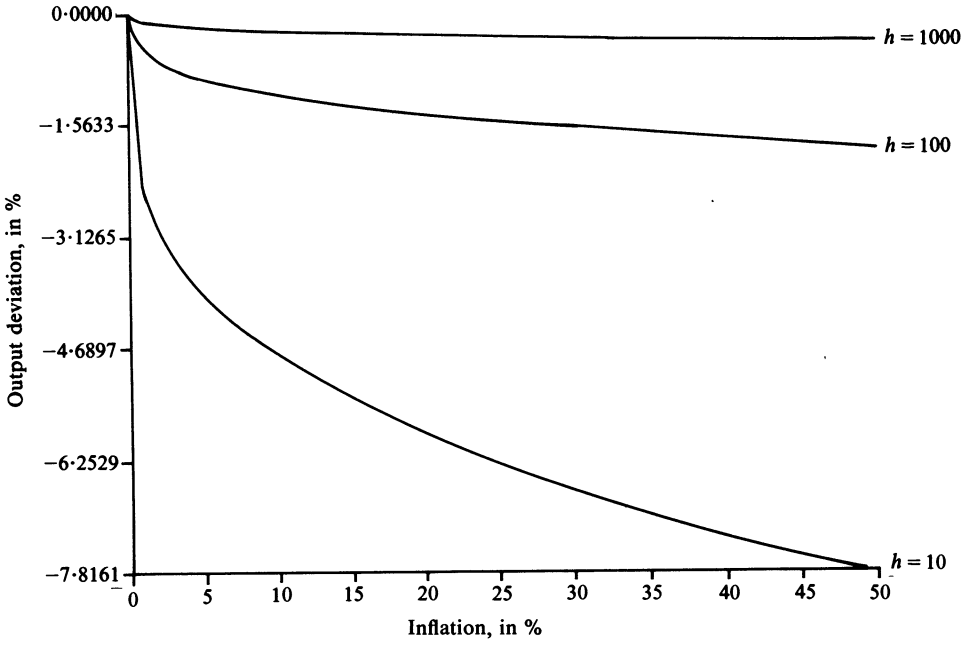

FIGURE 7

Inflation, output, and fixed costs. $c = 1$, $\beta = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p)^{-\alpha}$, $\alpha = 4{\cdot}1$

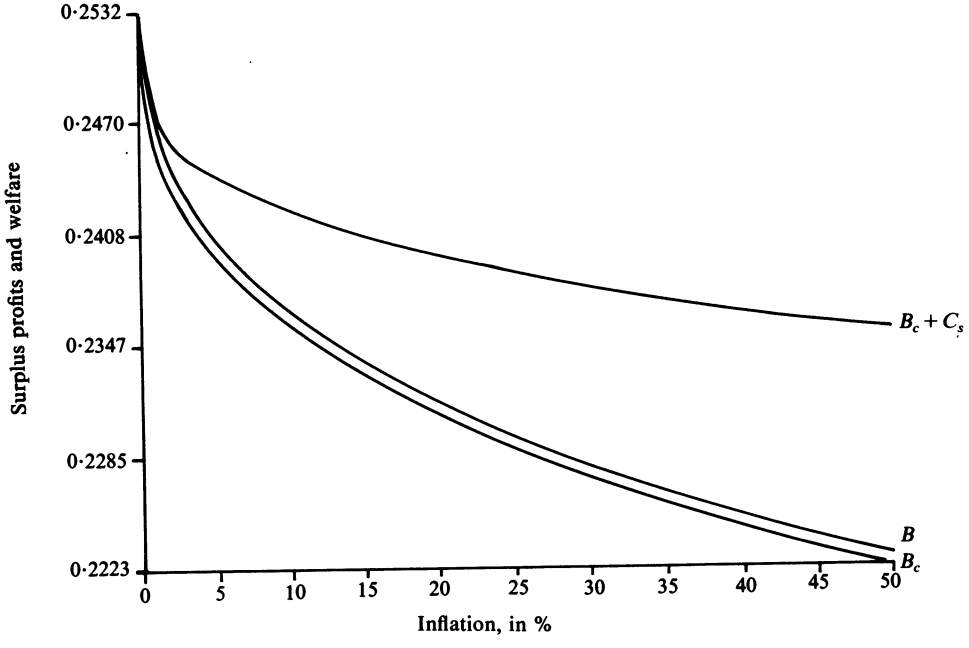

FIGURE 8(*a*)

Inflation and welfare. $c = 1$, $\beta = h = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p)^{-\alpha}$, $\alpha = 4{\cdot}1$. $B_c + C_s$ = gross consumer surplus, B_c = consumer surplus, net of search costs, $B - B_c$ = net profits, B = social welfare

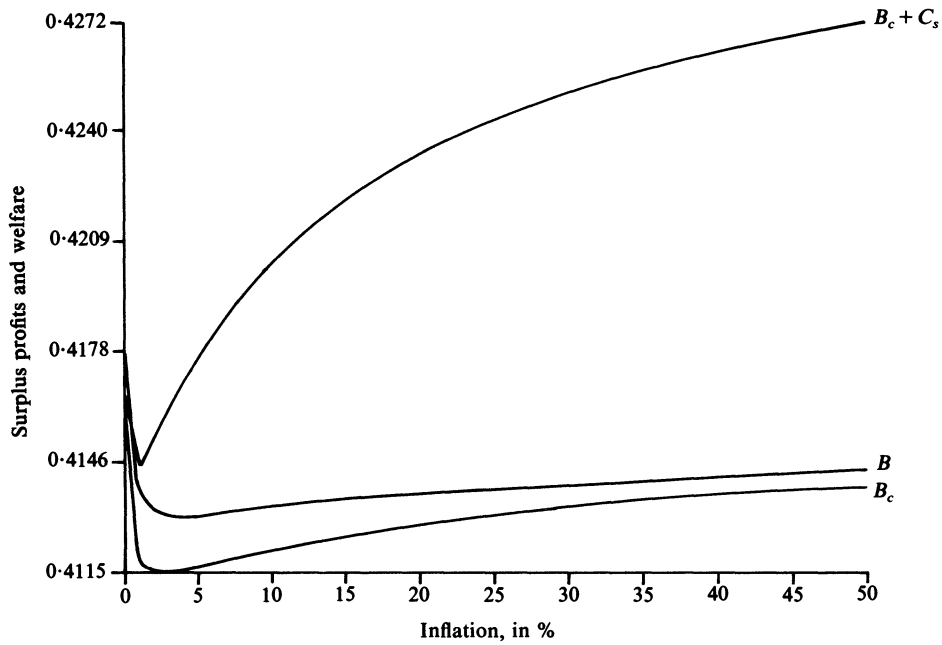

FIGURE 8(*b*)

Inflation and welfare. $c = 1$, $\beta = h = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p) = M - p$, $M = 8$. B_c, C_s, and B defined as on Figure 8(*a*)

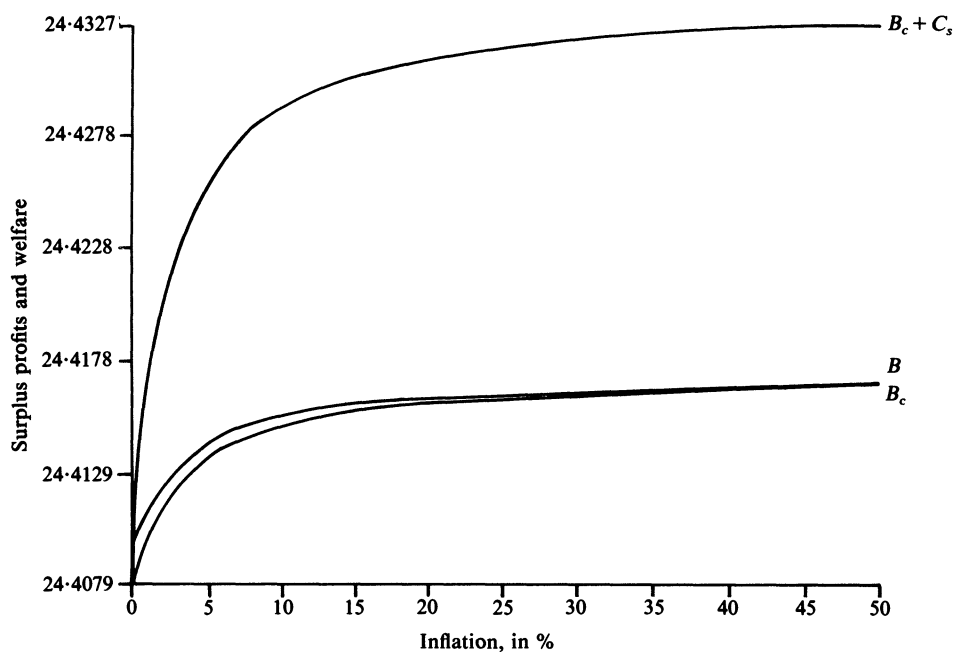

FIGURE 8(*c*)

Inflation and welfare. $c = 1$, $\beta = h = 10$, $\bar{\gamma} = 0{\cdot}10$, $\rho = 0{\cdot}05$, $z(p) = M - p$, $M = 8$. B_c, C_s, and B defined as on Figure 8(*a*)

$z(p)$: in Figure 8(a) a rise in g from 0% to 10% (respectively from 10% to 20%) decreases B by a substantial 6·6% (respectively 8·6%), but in Figure 8(b) it increases it by only 0·02% (respectively by 0·03%).

Result 5. *Whether inflation is beneficial or harmful to social welfare depends on preferences and market structure, and in particular, on whether search costs are low or high relative to consumer surplus. The variations of welfare most often mirror those of total output, but increased search expenditures may sometimes cause it to fall in spite of rising output.*

6. CONCLUSION

This paper has examined the functioning of a monopolistically competitive search market under inflation. It has formalized the idea that inflation increases the amount of resources spent on search, and more generally has developed a micro-theoretic framework to examine several channels through which inflation affects competition, output and welfare.

While inflation increases price dispersion and the resources devoted to search, it also has a long-run benefit, by reducing the number of firms and rent dissipation. But most importantly, it alters both the distribution of equilibrium prices and the distribution of buyers across prices. This effect can potentially alleviate or worsen the inefficiency of monopolistic pricing, as follows. In markets where search is inexpensive relatively to the surplus derived from the good, welfare will not fall very much, and may even rise with inflation, as most buyers take advantage of the increased price dispersion. When search is costly, on the contrary, inflation can significantly reduce output and welfare.

It is interesting to relate these results to those obtained by Benabou and Gertner (1990) for the effects of *unanticipated* inflation, i.e. of the price level's variance rather than its trend. Once it is recognized that agents can not only engage in signal extraction (as in Lucas (1973), Cuckierman (1979, 1983) or Hercowitz (1981)), but also acquire additional information through search, similar conclusions emerge: whether inflation uncertainty impairs or promotes efficiency depends on the size of informational costs.

Clearly, inflation's impact on search and competition is only one of its many consequences. Numerous other potential costs identified by Fischer and Modigliani (1978) lie outside the scope of this paper. Its aim, however, was only to examine a certain conventional wisdom about the distortions inflicted by inflation to the price system. In that respect, it shows that assessing the validity of such claims will in fact require empirical studies which go beyond simple measures of price dispersion, but pay particular attention to markups, entry or exit, and market structure.

APPENDIX

Proof of Proposition 2. The equilibrium profit function is (see equation (8) and footnote (14))

$$\Pi(p)=\frac{1}{\bar{\gamma}-\underline{\gamma}}\frac{\pi(p)}{\nu}[\min(V(p), V(\underline{r}))+\bar{\gamma}-\max(\underline{\gamma}, \bar{V}(S, s))] \tag{A1}$$

with $\pi(p)=(p-c)z(p)$ and $\bar{V}(S, s)=\Gamma(M)$. To simplify the notation, let $K\equiv\bar{\gamma}-\max(\underline{\gamma}, \bar{V}(S, s))$, and $\Delta(p)\equiv z(p)(V(p)+K)$, for all p. Finally, we denote the elasticity of monopoly demand $z(p)$ as $\alpha_z(p)\equiv -pz'(p)/z(p)$.

On the interval $(0, \underline{r}]$, $\Pi(p)$ is proportional to monopoly profits $\pi(p)$; given Proposition 1, it is therefore increasing, provided $\underline{r}\leqq p^m$.

On the interval $[\underline{r}, \bar{r})$, $D(p)$ is proportional to $\Delta(p)$ and $\Pi(p)\Pi_\Delta(p)\equiv(p-c)\Delta(p)$, which will now be shown to be strictly quasi-concave on its support $[c, \bar{r}]\subset[c, M]$. There are two cases to consider.

Case 1. When $K<0$, or $\bar{r}<M$, we show the stronger property that $\Delta(p)$ has increasing elasticity $\alpha_\Delta(p)$. Proposition 1 will then yield strict quasi-concavity of $\Pi_\Delta(p)$. On $(0, \bar{r})$ $\Delta(p)$ has elasticity:

$$\alpha_\Delta(p)=\alpha_z(p)+\frac{pz(p)}{V(p)+K}>\alpha_z(p) \tag{A2}$$

and:

$$\alpha'_\Delta(p)=\alpha'_z(p)+\frac{z(p)(1-\alpha_z(p))(V(p)+K))+p\cdot z(p)^2}{(V(p)+K)^2} \tag{A3}$$

where we used the fact that:

$$\frac{d(p\cdot z(p))}{dp}=z(p)(1-\alpha_z(p)). \tag{A4}$$

But this same equality yields:

$$p\cdot z(p)-\bar{r}\cdot z(\bar{r})=-\int_p^{\bar{r}} z(u)(1-\alpha_z(u))du$$

$$=(V(\bar{r})+K)(1-\alpha_z(\bar{r}))-(V(p)+K)(1-\alpha_z(p))+\int_p^{\bar{r}}(V(u)+K)\cdot\alpha'_z(u)du.$$

The numerator of the second term in (A3) thus becomes:

$$z(p)\left[\bar{r}\cdot z(\bar{r})+\int_p^{\bar{r}}(V(u)+K)\cdot\alpha'_z(u)du-(V(\bar{r})+K)(\alpha_z(\bar{r})-1)\right]. \tag{A5}$$

By definition of $\bar{r}$ and K, $\bar{r}<M$ implies $V(\bar{r})+K=0$, so the last term cancels out, while the remaining two are positive; hence the result.

Case 2. The case where $K\geqq 0$, or $\bar{r}=M$ is more difficult, because the last term in (A5) is generally negative. Consider marginal profits on $[0, M]$:

$$\Pi'_\Delta(p)=\Delta'(p)[p(1-1/\alpha_\Delta(p))-c]\equiv\Delta'(p)[h(p)-c] \tag{A6}$$

where $\alpha_\Delta(p)>\alpha_z(p)$ was defined in (A2). By Assumption 1, α_z is non-decreasing, so there exists an $m_z\in[0, M]$ such that $\alpha_z(p)\leqq 1$ on $(0, m_z]$ and $\alpha_z(p)\geqq 1$ on $[m_z, M)$; one of these intervals may be empty, but when $M=+\infty$, $\alpha_z(+\infty)>1$ implies $m_z<+\infty$.

On $(0, m_z)$, $\alpha_z(p)\leqq 1$ and (A4) imply that the function $p\cdot z(p)$ is non-decreasing. Therefore, by (A2) and (A3), α_Δ is strictly increasing on $[0, m_z]$, so there exists a unique $m_\Delta\in[0, m_z]$, such that $\alpha_\Delta(p)\leqq 1$ on $(0, m_\Delta]$ and $\alpha_\Delta(p)>1$ on $[m_\Delta, M)$.

On $[0, m_\Delta]$, $\alpha_\Delta(p)\leqq 1$, so $h(p)\leqq 0$ by (A6), or $\Pi'_\Delta(p)>0$. We shall now examine the variations of $h(p)$ on $[m_\Delta, M]$.

(i) On $(m_\Delta, m_z]$, $\alpha_\Delta(p)$ is increasing and $1-1/\alpha_\Delta(p)>0$, so $h(p)$ is increasing.

(ii) On $(m_z, M]$ (when it is not empty) $\alpha_\Delta(p)$ need not be monotonic but we shall prove that $h(p)$ is. Indeed:

$$\alpha_\Delta(p)^2\cdot h'(p)=\alpha_\Delta(p)^2-\alpha_\Delta(p)+p\cdot\alpha'_\Delta(p).$$

But multiplying (A3) by p and substituting in (A2) yields:

$$p\cdot\alpha'_\Delta(p)=p\cdot\alpha'_z(p)+(1-\alpha_z(p))(\alpha_\Delta(p)-\alpha_z(p))+(\alpha_\Delta(p)-\alpha_z(p))^2$$

$$\geqq(\alpha_\Delta(p)-\alpha_z(p))(1+\alpha_\Delta(p)-2\alpha_z(p))$$

so that (omitting the dependence on p):

$$\alpha_\Delta^2\cdot h'\geqq\alpha_\Delta^2-\alpha_\Delta+(\alpha_\Delta-\alpha_z)(1+\alpha_\Delta-2\alpha_z)$$

$$=\alpha_z(\alpha_z-1)+(\alpha_\Delta-\alpha_z)(\alpha_\Delta+\alpha_z-1+1+\alpha_\Delta-2\alpha_z)$$

$$=\alpha_z(\alpha_z-1)+(\alpha_\Delta-\alpha_z)(2\alpha_\Delta-\alpha_z)>0$$

since $\alpha_\Delta > \alpha_z > 1$. Thus $h(p)$ increases on $(m_\Delta, M]$ and is non-positive on $[0, m_\Delta]$. As a result, there exists a unique $p^* \in (m_\Delta, M]$, such that $h(p) < c$, i.e. $\Pi'_\Delta(p) > 0$ on $(0, p^*)$ and $h(p) > c$, i.e. $\Pi'_\Delta(p) < 0$ on $(p^*, M]$. Moreover, $\Pi'_\Delta(p^*) = 0$ if $p^* < M$, which always is the case when $M = +\infty$, because $\alpha_\Delta(+\infty) \geq \alpha_z(+\infty) > 1$ implies $h(+\infty) = +\infty$. ‖

Proof of Proposition 3. Let us first rewrite $W(S', s')$ by eliminating T' from (9). Denoting $\delta \equiv \rho/g$ and $u = S'e^{-gt}$, we have:

$$W(S', s') = \frac{\int_{s'}^{S'} \Pi(u) u^{-1+\delta} du - \beta g S'^{\delta}}{g(S'^{\delta} - s'^{\delta})}. \tag{A7}$$

Straightforward algebra gives the first-order conditions (FOC) for an interior optimum. Given the possible kink at $\bar{r}$ when $\bar{r} = M$, the most general condition for S' is: $\partial W(S'^{-}, s')/\partial S' \geq 0 \geq \partial W(S'^{+}, s')/\partial S'$, with equality unless $S' = \bar{r} = M$. But for $S' > \bar{r}$, $\partial W(S', s')/\partial S' = -\delta S'^{\delta-1}(\beta + W(S', s'))/(S'^{\delta} - s'^{\delta}) < 0$. Therefore $S' > \bar{r}$ is never optimal, and the first-order condition is simply $\partial W(S'^{-}, s')/\partial S' \geq 0$, i.e. $\Pi(S') - \Pi(s') \geq \rho\beta$, with equality unless $S' = \bar{r} = M$. Since $\Pi(S') \geq \rho(W + \beta) > 0$ requires $S' \leq \bar{r} \leq M$, this is equivalent to (13). The property that the FOC have at most one solution, and that, if it exists, it satisfies the second-order conditions and characterizes the optimum, follows from Sheshinski and Weiss (1977) and Zinde-Walsh (1987). ‖

Proof of Theorem 2. Given ν, an equilibrium (S, s) is fixed point of firms' best reply functions. With endogenous entry, however, it is a solution to (24)–(25), which corresponds to a fixed point of a somewhat different mapping, with a less straightforward economic interpretation.

1. *Preferences with bounded support*:
Assume first that $M < +\infty$, so that the non-empty set:

$$K \equiv \{(S, s) \in R^2 \mid c \leq s \leq p^m, s \leq S \leq M\} \tag{A8}$$

is not only convex but also compact. For all (S, s) in K, define as before the average surplus $\bar{V}(S, s)$ by:

$$\bar{V}(S, s) \equiv \int_s^S \frac{V(u)}{u} \cdot \frac{du}{\mathrm{Ln}(S/s)} \tag{A9}$$

for $s < S$, and by $\bar{V}(s, s) = V(s)$ (l'Hôpital's rule). Clearly:

$$V(S) \leq \bar{V}(S, s) \leq V(s) \tag{A10}$$

with strict inequality unless $S = s$. Define now, for all (S, s) in K and any $p \in [c, M]$:

$$\Pi_{(S, s)}(p) \equiv \pi(p) \cdot \max\{V(u) + \bar{\gamma} - \bar{V}(S, s), 0\} \tag{A11}$$

which is, up to a constant, the profit function of a firm in an (S, s) equilibrium. From Proposition 1, we know that $\Pi_{(S, s)}(p)$ is strictly quasi-concave (and clearly continuous) on its support, which is $[c, \bar{r}]$. Note that if $s < M$, $\Pi_{(S, s)}(p) > 0$ for p just above s, while if $s = M$, then $S = s = M$ so $\Pi_{(S, s)}(p) = \bar{\gamma}\pi(p) > 0$ for p just above c. Thus $\Pi_{(S, s)}$ never has trivial support ($\bar{r} > c$).

Next, define for all (S, s) in K:

$$J(S, s) = \int_s^S \Pi_{(S, s)}(u) \frac{u^{\delta-1}}{(S^{\delta} - s^{\delta})/\delta + g\beta S^{\delta}/h} du. \tag{A12}$$

$J(S, s)$ is a weighted average of a firm's discounted profits in an (S, s) equilibrium, i.e. of $\rho W(S, s)$, given by (12), and of h; with entry, $\rho W(S, s) = h$, so $J(S, s) = \rho W(S, s) = h$.

Denoting by $p^*_{(S, s)} \leq p^m$ the unique maximum of the function $\Pi_{(S, s)}$, and by $\Pi^*_{(S, s)}$ its maximal value, we have:

$$0 \leq J(S, s) < \Pi^*_{(S, s)} \cdot \frac{(S^{\delta} - s^{\delta})/\delta}{(S^{\delta} - s^{\delta})/\delta + g\beta S^{\delta}/h} < \frac{\Pi^*_{(S, s)}}{1 + g\beta/h} \tag{A13}$$

with the first inequality being strict unless $S = s$ (since $S < +\infty$). The strict quasi-concavity of $\Pi_{(S, s)}(p)$ and (A13) imply:

$$\begin{aligned}
&\forall (S, s) \in K, \exists!(s', S') \in [c, p^*_{(S, s)}) \times (p^*_{(S, s)}, M] \subset K \text{ such that} \\
&\Pi_{(S, s)}(s') = J(S, s); \\
&\Pi_{(S, s)}(S') \geq (1 + \rho\beta/h) J(S, s); \\
&[\Pi_{(S, s)}(S') - (1 + \rho\beta/h) J(S, s)](M - S) = 0.
\end{aligned} \tag{A14}$$

Moreover, note that: $s' > c$ unless $J(S, s) = 0$, i.e. unless $S = s$. We shall denote the solution to (A14), given (S, s), by $(S', s') = \Psi(S, s)$.

The functions appearing in (A14) are clearly continuous in $(S, s; S', s')$. Together with the *uniqueness* of the solution (S', s'), this implies that Ψ is continuous in (S, s) on K. Indeed, if $(S_n, s_n)_{n\in N}$ converges to $(S, s) \in K$, then the corresponding sequence $(S'_n, s'_n)_{n\in N}$ is in the compact set K, so it must have at least one accumulation point (S'', s''). But writing down (A14) for $(S_n, s_n; S'_n, s'_n)$ and taking limits implies that $(S, s; S'', s'')$ verifies (A14). The uniqueness of the solution, given (S, s) implies that any such accumulation point (S'', s'') must equal (S', s'); thus $(S'_n, s'_n)_{n\in N}$ converges to (S', s').

The function Ψ, which maps the convex, compact set K into itself, must have a fixed point $(S^*, s^*) = \Psi(S^*, s^*) = (S^{*\prime}, s^{*\prime})$. Moreover, by (A14), $s^{*\prime} < p^*_{(S,s)} < S^{*\prime}$, so: (i) $s^* < p^m$; (ii) $s^* < S^*$, hence $J(S^*, s^*) > 0$ and $s^* > c$. This ensures that $\nu \in (0, +\infty)$. Similarly, if $z(M) = 0$, then necessarily $S < M$. This concludes the proof in this case.

2. *Preferences with unbounded support*:

Assume now that $z(p)$ and $V(p)$ have support $(0, +\infty)$. For each finite $M > p^m$, consider the truncated demand function $z_M(p)$ (or surplus $V_M(p)$) which coincides with $z(p)$ (or $V(p)$) on $(0, M]$, and is zero afterwards. Since z_M still satisfies Assumption 1, the above result guarantees the existence of an equilibrium (S^*_M, s^*_M) for these modified preferences. The equilibrium (S^*, s^*) for the infinite-support problem will be constructed as a limit of (S^*_M, s^*_M) for a sequence of values of M going to $+\infty$. In order to do so, it must be proved that S^*_M remains bounded even as M tends to $+\infty$, shall start by proving:

Lemma 1. *There exists $B > p^m$, such that for all $s \in [c, p^m]$, and all $S > B$:*

$$\frac{J(S, s)}{\Pi_{(S,s)}(S)} > \frac{1}{1+\rho\beta/h}. \tag{A15}$$

Moreover, the same inequality holds, when $B < S < M$, for the functions $\Pi^M_{(S,s)}(p)$ and $J^M(S, s)$ associated by (A9)-(A11)-(A12) *to any finite truncation of preferences at $M > B$.*

This result will prove that for all $M > B$, $S^*_M \leqq B$, or else the second condition of (A14), for the corresponding functions $\Pi^M_{(S,s)}(p)$ and $J^M(S, s)$, would not hold for $S^{*\prime}_M = S^*_M$. Note that since $V(p)$ is decreasing and $s > 0$:

$$\frac{J(S, s)}{\Pi_{(S,s)}(S)} = \int_s^S \frac{\pi(u)(V(u)+\bar{\gamma}-\bar{V}(S,s))}{\pi(S)(V(p)+\bar{\gamma}-\bar{V}(S,s))} \cdot \frac{\delta u^{\delta-1}}{S^\delta(1+\rho\beta/h)-s^\delta}\, du$$

$$> \int_s^S \frac{\pi(u)}{\pi(S)} \cdot \frac{\delta u^{\delta-1} du}{S^\delta(1+\rho\beta/h)}.$$

The same holds true in any truncated problem, so that it suffices to prove

Lemma 2. *There exists $B > p^m$, such that for all $s \in [c, p^m]$ and all $S > B$:*

$$\int_s^S \frac{\pi(u)}{\pi(S)} \frac{\delta u^{\delta-1}}{S^\delta}\, du > 1. \tag{A16}$$

Moreover, the same inequality also holds, for $B < S < M$, for the function $\pi^M(p) = (p-c)z^M(p)$ associated to any truncation of preferences at $M > B$.

Proof. For $M > S > B$, the left-hand-side of (A16) remains unchanged when preferences are truncated at M, so that attention can be confined to the untruncated case. Under Assumption 1, there exists $A > p^m$ and $\alpha > 1$ such that, for all $p \geqq A$: $z'(p)/z(p) > -\alpha/p$. Integrating over $[u, S]$ for $u < S$:

$$\frac{z(S)}{z(u)} < \left[\frac{S}{u}\right]^{-\alpha} \tag{A17}$$

The left-hand-side of (A16) is therefore bounded from below by:

$$\int_A^S \frac{(u-c)}{(S-c)} \cdot \frac{u^{\delta-\alpha+1}}{S^{\delta-\alpha}}\, du = \frac{\delta}{s^{\delta-\alpha}(S-c)} \left[\frac{S^{\delta-\alpha+1} - A^{\delta-\alpha+1}}{\delta-\alpha+1} - \frac{S^{\delta-\alpha} - A^{\delta-\alpha}}{\delta-\alpha}\right] \tag{A18}$$

where we have assumed, for now, that $\delta - \alpha \notin \{0, -1\}$.

Case 1. $\delta - \alpha + 1 > 0$; as S tends to $+\infty$, the terms in $S^{\delta-\alpha+1}$ dominate the above expression, which therefore tends to $\delta/(\delta-\alpha+1) > 1$, hence the result.

Case 2. $\delta-\alpha+1<0$: the term in brackets is always positive, and in the limit both $S^{\delta-\alpha+1}$ and $S^{\delta-\alpha}$ tend to zero; since the first term is equivalent to $\delta/S^{\delta-\alpha+1}$, the above expression tends to $+\infty$ with S, hence the result.

Case 3. $\delta-\alpha+1=0$; the right-hand side of (A18) must then be replaced by $[\delta S/(S-c)][Ln(S/A)+c.(1/A-1/S)]$, which also tends to $+\infty$ with S.

Case 4. $\alpha-\delta=0$; the right-hand side of (A18) must then be replaced by $\delta[(S-A)/(S-c)-c\cdot \mathrm{Ln}(S/A)/(S-c)]$, which also tends to $+\infty$ with S. ‖

This concludes the proof of Lemma 2, in all cases. Consider now a sequence of truncations at $M=n$, for $n\in N$, $n>B$, with corresponding equilibria (S_n^*, s_n^*). By definition, $(S_n^*, s_n^*; S_n^*, s_n^*)$ satisfy (A14), for the functions $\Pi^n_{(S,s)}(p)$ and $J^n(S,s)$. By Lemma 1, the sequence $(S_n^*, s_n^*)_{n\in N}$ remains in the compact set $[c, p^m]\times[p^m, B]$, so there exists a subsequence which converges to some limit (S^*, s^*); taking limits in (A14) for that subsequence easily yields the result that $(S^*, s^*; S^*, s^*)$ satisfies (A14) for the untruncated functions $z(p)$ and $V(p)$, so that (S^*, s^*) is an equilibrium for the original preferences with unbounded support. ‖

Proof of Proposition 4. Define the function $\varphi(u)=\varphi_{(S,s)}(u)\equiv\pi(u)[V(u)+\bar\gamma-\bar V(S,s)]$; the subscript (S,s) will be omitted when no confusion results. Denote also $\Psi(u)\equiv\pi(u)[V(u)+\bar\gamma-V(p_*)]$, where p_* is the equilibrium price when $\beta=0$; Ψ corresponds to Π_* defined in the main text. By definition, $\varphi(p_*)=\Psi(p_*)=\bar\gamma.\pi(p_*)$ and $\Psi'(p_*)=0$. We shall abbreviate $\Psi(p_*)$ as Ψ_*, $\Psi''(p_*)$ as Ψ''_*, $V(p_*)$ as V_*, etc. Let us start by rewriting (24)–(25) as:

$$\varphi(S)-\varphi(s)=\varepsilon\cdot\varphi(s) \tag{A19}$$

$$\int_s^S[\varphi(u)-\varphi(p_*)]u^{\delta-1}du=\varepsilon\cdot\varphi(s)\cdot S^\delta/\delta+[\varphi(s)-\varphi(p_*)](S^\delta-s^\delta)/\delta \tag{A20}$$

where $\varepsilon\equiv\rho\beta/h\ll 1$. We shall neglect all terms of order higher than ε, denoted as $o(\varepsilon)$; the system then becomes:

$$\varphi(S)-\varphi(s)=\varepsilon\cdot\Psi_*+o(\varepsilon) \tag{A21}$$

$$\int_s^S[\varphi(u)-\varphi(p_*)]u^{\delta-1}du-\Psi_*\cdot p_*^\delta\cdot\varepsilon/\delta+[\varphi(s)-\varphi(p_*)](S^\delta-s^\delta)/\delta. \tag{A22}$$

Moreover, $\varphi(u)=\Psi(u)-\pi(u)[\bar V(S,s)-V_*]$. Next, define $X\ll 1$ and $x\ll 1$ by $S=p_*(1+X)$, $s=p_*(1-x)$, and expand $\bar V(S,s)-V_*$, $\varphi(u)-\varphi(p)$, and finally the whole system (A21)–(A22) up to the third order in (X,x); higher-order terms are denoted $o(3)$. Tedious but straightforward calculations (available from the author upon request) lead to:

$$(X+x)[A_1(X-x)+A_2(X^2-Xx+x^2)+A_3(X-x)^2]+o(3)=\varepsilon\cdot\Psi_*+o(\varepsilon) \tag{A23}$$

$$(X+x)[B_1(2x^2-X^2+Xx)+B_2X(X-x)]+o(3)=\varepsilon\cdot\Psi_*/\delta+o(\varepsilon) \tag{A24}$$

where

$$A_1\equiv p_*^2\cdot\Psi''_*/2-\phi_1\cdot p_*\cdot\pi'_* \qquad A_2\equiv p_*^3\cdot\Psi'''_*/6-\phi_2\cdot p_*\cdot\pi'_*$$

$$A_3\equiv-\phi_1\cdot p_*^2\cdot\Psi''_*/2 \qquad B_1=-p_*^2\cdot\Psi''_*/6 \qquad B_2\equiv-\phi_1\cdot p_*\cdot\pi'_*$$

$$\phi_1=-p_*\cdot z_*/2 \qquad \phi_2\equiv 2p_*\cdot z_*-p_*^2\cdot z'_*$$

Now let η be the highest of the orders of X and x in ε: $X=\omega\cdot\varepsilon^\eta+o(\varepsilon^\eta)$, $x=\sigma\cdot\varepsilon^\eta+o(\varepsilon^\eta)$, with $\omega+\sigma>0$. The system becomes:

$$\varepsilon^{2\eta}\cdot(\omega+\sigma)[A_1(\omega-\sigma)+A_2(\omega^2-\omega\sigma+\sigma^2)+A_3(\omega-\sigma)^2]+o(\varepsilon^{2\eta})=\varepsilon\cdot\Psi_*+o(\varepsilon) \tag{A25}$$

$$\varepsilon^{3\eta}\cdot(\omega+\sigma)[B_1(2\sigma^2-\omega^2+\omega\sigma)+B_2\omega(\omega-\sigma)]+o(\varepsilon^{3\eta})=\varepsilon\cdot\Psi_*/\delta+o(\varepsilon) \tag{A26}$$

Equation (A26) implies that $\eta=1/3$. Equation (A25) then requires that $\omega-\sigma=0$ (and that $X-x$ be of order ε), so that (A26) becomes $4\cdot B_1\cdot\sigma^3=\Psi_*/\delta$, which proves (27) in Proposition 4.

We now turn to C^s, assuming first $\bar{r} < M$. With $\gamma = 0$ (26) becomes:

$$\bar{\gamma}[C_s + \bar{\gamma}/2] = \int_s^{\bar{r}} z(r)\Gamma(r)dr$$

$$= [(\bar{V}(S,s) - V(r))\Gamma(r)]_s^{\bar{r}} - \int_s^{\bar{r}} (\bar{V}(S,s) - V(r))z(r)F(r)dr$$

$$= (\bar{V}(S,s) - V(\bar{r})) \cdot \bar{\gamma} - [(\bar{V}(S,s) - V(r))^2/2]_S^{\bar{r}}$$

$$- \int_s^S (\bar{V}(S,s) - V(r))z(r)F(r)dr$$

But

$$\bar{V}(S,s) = \Gamma(M) = \Gamma(\bar{r}) + \int_{\bar{r}}^M z(r)dr = \bar{\gamma} + V(\bar{r}),$$

so

$$\bar{\gamma} \cdot C_s = (\bar{V}(S,s) - V(S))^2/2 - \int_s^S (\bar{V}(S,s) - V(r))z(r)F(r)dr. \tag{A27}$$

When $\bar{r} = M$, similar calculations show that (A27) remains unchanged. Finally, Taylor expansions of second order in $x = X$ (using the previous results) yield:

$$(\bar{V}(S,s) - V(S))^2/2 = z_*^2 \cdot p_*^2 \cdot x^2/2 + o(x^2)$$

$$\int_s^S (\bar{V}(S,s) - V(r))z(r)F(r)dr = z_*^2 \cdot p_*^2 \cdot x^2/3 + o(x^2)$$

hence the result in (27) ‖

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