Image Versus Information: Changing Societal Norms and Optimal Privacy

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Online Appendix

In this Online Appendix, we offer the analyses relevant for Section VI, and pertaining to an extension of our model where the Principal determines the noise with which each agent's action is observed by others (discussed in Section I.A).

B Analysis of Norms Shaping Laws in VI.A

Agent i's non-reputational payoffs in period 1 and 2 are:

$$U_i^1(v_i, \theta, w, a_i) = (v_i + \theta)a_i + (w + \theta)\bar{a} - \frac{a_i^2}{2}, \tag{B.1}$$

$$U_i^2(v_i, \theta, w, a^*) = (w + \theta)\bar{a} - \frac{(a^*)^2}{2}, \tag{B.2}$$

and he solves: $\max_{a_i} \mathbb{E}\left[U_i^1 + x\,\mu_i\left(R(a_i, \theta_i, \mu_i) - \bar{v}\right) + \delta\, U_i^2\right]$. Agents thus derive no intrinsic satisfaction from compulsory contributions; the analysis would remain the same if they did, however. The same steps as in Proposition 1 lead again to $a_i = v_i + \rho\,\theta_i + (1-\rho)\bar{\theta} + x\xi(x)\mu$, with $\xi(x)$ unchanged from (19). Turning now to the Principal, her objective function is $E[V^1 + \delta V^2]$, where $\tilde{R} \equiv \int_0^1 \mathbb{E}\left[v_i | a, \bar{a}\right]\, dj$ and

$$\begin{aligned} V^1 &= \lambda\left(\alpha\int_0^1 (v_i + \theta)a_i\, d_i + (w + \theta)\bar{a} + \tilde{\alpha}\int_0^1 x\,\mu_i\left(\tilde{R}(a_i, \bar{a}) - \bar{v}\right) di - \int_0^1 \frac{a_i^2}{2}\, di\right) \\ &\quad + (1 - \lambda)b(w + \theta)\bar{a}, \\ V^2 &= \lambda\left((w + \theta)a^* + \tilde{\alpha}\int_0^1 x\,\mu_i\left(\tilde{R}(a_i, \bar{a}) - \bar{v}\right) di - \int_0^1 \frac{(a^*)^2}{2}\, di\right) + (1 - \lambda)b(w + \theta)a^*. \end{aligned}$$

Maximizing $\mathbb{E}\left[V^2 | \bar{a}, \theta_P\right]$ over a^* leads to

$$\begin{aligned} 0 &= \lambda\left((w + \mathbb{E}\left[\theta | \bar{a}, \theta_P\right]) - a^*\right) + (1 - \lambda)b(w + \mathbb{E}\left[\theta | \bar{a}, \theta_P\right]), \text{ or} \\ a^* &= \frac{w\varphi}{\lambda} + \frac{\varphi}{\lambda}\mathbb{E}\left[\theta | \bar{a}, \theta_P\right]. \end{aligned} \tag{B.3}$$

If, after choosing x, will learn the realized value of θ or μ (allowing her to invert $\bar{a}$ and learn θ perfectly), this reduces to $a^* = [w\varphi + \varphi\,\theta]/\lambda$ and substituting into the objective function yields

$$\begin{aligned} E\tilde{V}(x) =& \lambda\Big[\alpha\left((\bar{v}+\bar{\theta})\bar{a} + s_v^2 + \rho\sigma_\theta^2 + \delta\frac{\varphi}{\lambda}\sigma_\theta^2\right) + \left((w+\bar{\theta})(\bar{\bar{a}}+\delta\bar{a}^*) + \rho\sigma_\theta^2 + \delta\frac{\varphi}{\lambda}\sigma_\theta^2\right) \\ & + (1+\delta)\tilde{\alpha}\frac{x^2\xi(x)^2}{(1+\delta)}s_\mu^2 - \frac{1}{2}[\bar{\bar{a}}^2 + s_v^2 + \rho^2(\sigma_\theta^2 + s_\theta^2) + x^2\xi(x)^2(\sigma_\mu^2 + s_\mu^2)] \\ & -\frac{\delta}{2}\left((\bar{a}^*)^2 + \left(\frac{\varphi+\lambda\alpha}{\lambda}\right)^2\sigma_\theta^2\right)\Big] + (1-\lambda)\left[b(w+\bar{\theta})(\bar{\bar{a}}+\delta\bar{a}^*) + \rho\sigma_\theta^2 + \delta\frac{\varphi}{\lambda}\sigma_\theta^2\right], \quad \text{(B.4)} \end{aligned}$$

where: $\bar{a}^* \equiv \left(w\varphi + \varphi\bar{\theta}\right)/\lambda$. The first order condition is

$$\begin{aligned} 0 = \lambda & \left[\alpha(\bar{v}+\bar{\theta})\bar{\mu}\beta'(x) + (w+\bar{\theta})\bar{\mu}\beta'(x) + 2\tilde{\alpha}s_\mu^2 x\xi(x)\beta'(x) - (\bar{v}+\bar{\theta}+x\xi(x)\bar{\mu})\bar{\mu}\beta'(x)\right. \\ & \left. - x\xi(x)(\sigma_\mu^2 + s_\mu^2)\beta'(x)\right] + (1-\lambda)\left[b(w+\bar{\theta})\bar{\mu}\beta'(x)\right], \end{aligned}$$

which leads to:

$$x^* = \frac{\bar{\mu}\omega}{\xi(x^*)\,\lambda\,\left(\bar{\mu}^2 + \sigma_\mu^2 + (1-2\tilde{\alpha})s_\mu^2\right)}. \quad \text{(B.5)}$$

When the Principal does not observe θ (or μ), finally, Proposition 14 shows that the expectation in (B.3) remains unchanged: $E[\theta|\theta_P, \bar{a}] = [1-\gamma(x)]\bar{\theta}_P + \gamma(x)\hat{\theta}$, with $\bar{\theta}_P$ still given by (21) and $\gamma(x)$ by (24). Hence, similarly to (27):

$$EV(x) = E\tilde{V}(x) - \frac{\delta}{2}\frac{\varphi^2}{\lambda}\sigma_P^2[1-\gamma(x)].$$

Noting, as in the Proof of Proposition 6, that $\gamma'(x) = -(2\sigma_\mu^2/\rho^2\sigma_P^2)\beta(x)\beta'(x)\gamma(x)^2$, and substituting into the first-order condition for $EV(x)$ yields

$$x^* = \frac{\omega\,\bar{\mu}}{\xi(x^*)\,\left(\lambda\left(\bar{\mu}^2 + \sigma_\mu^2 + (1-2\tilde{\alpha})s_\mu^2\right) + \frac{\delta}{\lambda}\left(\frac{\varphi\sigma_\mu\,\gamma(x^*)}{\rho}\right)^2\right)}. \quad \text{(B.6)}$$

Given the similarity with the benchmark expressions, the same comparative statics follow.

C Analysis of Norms Shaping Incentives in VI.B

The Principals' second-period policy is now to set an incentive rate y', under which agents contribute a second time, rather than constraining them to a legal mandate a^*. For simplicity we assume here that there is no reputational payoff in the second period (no period 3 in which agents would play some continuation game were reputation was valuable). As to the Principal, she again has intertemporal objective function $V^1 + \delta V^2$, with components now given by:

$$V^1 = \lambda \left(\alpha \int_0^1 (v_i + \theta) a_i \, d_i + (w + \theta)\bar{a} + \tilde{\alpha} \int_0^1 x \, \mu_i \left(\tilde{R}(a_i, \bar{a}) - \bar{v} \right) di - \int_0^1 \frac{a_i^2}{2} \, di \right)$$
$$+ (1 - \lambda) b(w + \theta)\bar{a}, \tag{C.1}$$
$$V^2 = \lambda \left(\alpha \int_0^1 (v_i + \theta)(a_i' - y') \, d_i + (w + \theta + y')\bar{a}' - \int_0^1 \frac{(a_i')^2}{2} \, di \right)$$
$$+ (1 - \lambda) \left[b(w + \theta) - (1 + \kappa) y' \right] \bar{a}', \tag{C.2}$$

where "primes" denote second-period actions and, as in the case of first-period incentives: (i) the Principal faces a shadow cost $(1+\kappa)$ per unit of funds; (ii) agents derive intrinsic satisfaction only from the portion of their contributions a_i' that is not directly driven by the incentive y'.

Using the notation $a_i(x)$ to denote equilibrium contributions in the baseline model, given by (9), it is clear, given our assumptions, that:

(a) In the first period, agents contribute again the very same $a_i(x)$, for every realization of their (v_i, θ_i, μ_i). Thus both the informativeness $\xi(x)$ of actions about individual types and the informativeness $\gamma(x)$ of aggregate compliance $\bar{a}(x)$ about θ remain unchanged.

(b) In the second period, since agents no longer have any reputational concerns (equivalently, $x' \equiv 0$) but now face material incentives y, each of them contributes $a_i'(y) \equiv a_i(0) + y'$.

Let us again focus (for simplicity only) on the case where $\lambda = 1/2$. The problem of the Principal in period 2 is to choose y' to maximize $E[V^2 | \theta_P, \bar{a}]$:

$$\max_{y'} E \left[\alpha \int_0^1 (v_i + \theta) a_i(0) di + (w + \theta)(1 + b)(\bar{a}(0) + y') - \int_0^1 \frac{(a_i(0) + y')^2}{2} di - \kappa y (\bar{a}(0) + y') | \theta_P, \bar{a} \right]$$

The first order condition yields the optimal level of incentives

$$y' = \frac{w(1 + b) - (1 + \kappa)(\bar{v} + (1 - \rho)\bar{\theta})}{1 + 2\kappa} + \frac{(1 + b) - \rho(1 + \kappa)}{1 + 2\kappa} E[\theta | \theta_P, \bar{a}]. \tag{C.3}$$

Quite intuitively, it is increasing in her posterior $E[\theta | \theta_P, \bar{a}]$, but with a slope that declines with the shadow cost of funds κ.

Consider now period 1. As observed above, since reputation is based only on actions and that period, $a_i(x), \xi(x)$ and $\gamma(x)$ all remain unchanged from the benchmark model, so there only remains to solve for the optimal x. As usual, consider first the case in which θ (or μ) is observed by the Principal at the beginning of period 2. Then, (C.3) becomes:

$$y' = \frac{w(1 + b) - (1 + \kappa)[\bar{v} + (1 - \rho)\bar{\theta}]}{1 + 2\kappa} + \frac{[(1 + b) - \rho(1 + \kappa)]}{1 + 2\kappa} \theta. \tag{C.4}$$

The Principal's objective function in period 2 is thus independent of x, implying that the optimal x maximizes $E[V^1]$ and is therefore given (A15), in which we set $\lambda = 1/2$:

$$\tilde{x} = \frac{2\bar{\mu}\omega}{\xi(\tilde{x})[\bar{\mu}^2 + \sigma_\mu^2 + (1 - 2\tilde{\alpha})s_\mu^2]}. \tag{C.5}$$

Suppose, finally, that the Principal does not observe either θ or μ, and thus uses $\bar{a}$ and θ_P to update her prior. The optimal incentive rate in period 2 is given by (C.3), in which $E[\theta|\theta_P] = \bar{\theta}_P$, $\gamma(x)$, $E[\theta|\theta_P, \bar{a}] = (1 - \gamma(x))\bar{\theta}_P + \gamma(x)\hat{\theta}$ and $V(\Delta) = \sigma_P^2(1 - \gamma(x))$ all remain unchanged from the baseline model. Note that, as a result, y' rises with the observed $\bar{a}$, but with a slope that decreases in κ. Consequently, with $\lambda = 1/2$ we have

$$EV(x) = E\tilde{V}(x) - \frac{\delta}{4}\left[\frac{(1+b) - \rho(1+\kappa)}{1+2\kappa}\right]^2 \sigma_P^2(1 - \gamma(x)), \tag{C.6}$$

which leads to

$$\frac{\partial EV(x)}{\partial x} = \frac{\partial E\tilde{V}(x)}{\partial x} - \frac{\delta}{2}\left(\frac{[(1+b) - \rho(1+\kappa)]\sigma_\mu\gamma(x)}{\rho(1+2\kappa)}\right)^2 x\xi(x)\beta'(x)$$

and the equation defining the optimal x^*

$$x^* = \frac{2\omega\bar{\mu}}{\xi(x^*)\left[\bar{\mu}^2 + \sigma_\mu^2 + (1-2\tilde{\alpha})s_\mu^2 + \delta\left(\frac{[(1+b)-\rho(1+\kappa)]\sigma_\mu\gamma(x)}{\rho(1+2\kappa)}\right)^2\right]}. \ \blacksquare \tag{C.7}$$

D Analysis of Noisy Observability in Section I.A

In the main text, we described how our results follow from an alternative specification of publicity in which the Principal determines the noise with which each individual contribution is observed. Specifically, suppose that when i contributes a_i, others observe $\hat{a}_i = a_i + \varepsilon_i$ and $\varepsilon_i \sim N(0, s_\varepsilon^2/x^2)$, where x is chosen by the Principal. To show that our results apply to both private and common values, let $\hat{\rho} = 1$ for a private-values environment and $\hat{\rho} = \sigma_\theta^2/(\sigma_\theta^2 + s_\theta^2)$ for the common-values case.

In this version of the model, the return to image is

$$\hat{\xi}(x) = \frac{s_v^2}{\hat{\xi}(x)^2 s_\mu^2 + s_\varepsilon^2/x^2 + s_v^2 + \hat{\rho}^2 s_\theta^2}. \tag{D.1}$$

Moreover,

$$a_i = \hat{\xi}(x)\mu_i + v_i + \hat{\rho}\theta_i + (1 - \hat{\rho})\bar{\theta}$$

so that $\hat{\xi}(x)$ also represents the impact on visibility on contributions. This contrasts with our baseline model in which this term was $\beta(x) \equiv x\xi(x)$. Note from (D.1) that $\hat{\xi}(x)$ is *increasing* in x (just as $\beta(x)$ did in the previous version of the model).

The Principal's choice of a_P is unchanged and remains just as before. Using an analysis identical to that of our baseline model, one can show that the optimal level of noise x^* solves:

$$\hat{\xi}(x^*) = \frac{\omega\bar{\mu}}{\lambda(\bar{\mu}^2 + \sigma_\mu^2 + (1-2\tilde{\alpha})s_\mu^2) + \frac{1}{(1-\lambda)k_P}\left(\frac{\varphi\sigma_\mu\gamma(x^*)}{\hat{\rho}}\right)^2}. \tag{D.2}$$

The only difference between (D.2) and (32) is that $\xi(x)$ has replaced $\beta(x)$. Because $\xi(x)$ and $\beta(x)$ are co-monotonic in all of the primitives, the comparative statics of this version of the model are identical to that of our baseline model.